# Probabilistic evaluation of column over-design factor for frame structures considering seismic base shear distribution of BNBC 

M. Sharfuddin ${ }^{1}$, Y.G. Zhao ${ }^{2}$, H. Idota ${ }^{3}$ and M. A. Ansary ${ }^{4}$<br>${ }^{1,3}$ Department of Architecture, Nagoya Institute of Technology, Nagoya, Japan<br>${ }^{2}$ Department of Architecture, Kanagawa University, Yokohama, Japan<br>${ }^{3}$ Depertment of Civil Engineering<br>Bangladesh University of Engineering and Technology, Dhaka, Bangladesh

Received 02 June 2010


#### Abstract

Frame structures are usually designed with a column overdesign factor (COF) to ensure yielding of all beams in flexure prior to possible yielding of columns during earthquakes, which is generally considered to be the preferable failure mode. In the present study, initially the failure modes of the multistory ductile frames structure grouping into three types: upper story collapse, middle story collapse and lower story collapse are investigated probabilistically applying first order reliability method (FORM) considering seismic base shear distribution of Bangladesh National Building Code (BNBC). Based on the investigations, the target COF values are proposed that ensures probabilistically the preferable entire beam hinging failure mode and avoid probabilistically the undesirable story collapse modes of the frame. The study has been conducted for different reliability level and the corresponding COF values are presented in this paper.


© 2010 Institution of Engineers, Bangladesh. All rights reserved.
Keywords: Frame structures, failure modes, probabilistic analysis, column over-design factor

## 1. Introduction

Earthquakes can cause great loss of life and property by destroying structures such as buildings, bridges and dams. The most destructive earthquakes are caused by seismic waves that reach to the earth surface at areas where man-made structures are located. For the safety of the buildings during earthquake shaking, the philosophy of strong column weak beam has widely been accepted by the structural designers and the researchers. In this seismic design concept, it is assumed that yielding of all beams in flexure will occur prior to possible yielding of columns which is considered to be the preferable failure
mode (Anderson and Gupta 1972; Clough and Penzine 1982; Park and Pauly 1975; Lee 1996).

To ensure that a frame structure collapses according to the preferable beam-hinging pattern, the columns of the structure that receive forces from the beams of a building structure are generally overdesigned with a COF value greater than one to make the columns relatively stronger than the beams. Different COF requirements have been addressed by different structural codes. The International Building Code (2006) and American Concrete Institute 318 code (2005) require that the ratio of nominal column to beam strengths for concrete structure should be greater than 1.2. The same ratio is provided for the concrete frames by the Bangladesh National Building Code (BNBC 1993). According to design provision of Japan (BCJ 2004) a minimum COF of 1.5 is suggested for cold-formed square tube structures in Japan and in seismic provision of structural steel building (ANSI/AISC 341-05) a COF value of 1.0 is suggested for steel structures. In other countries, such as New Zealand and Mexico, a COF ranging from 1.5 to 2.0 is adopted (Dooley \& Bracci 2001).

Many studies also have been conducted so far by the researchers in search of dominant collapse modes of the frames and designing strong column weak beam frames. Hibino and Ichinose (2005) presented a numerical study on the effect of column-to-beam strength ratio on the seismic energy dissipation of beams and columns in fish-bone-type steel moment frames. The major parameters considered are number of stories, strengths of columns, strengths of beams and ground motion. Findings of the study show that with the increase of the beam to column strength ratio the energy contributing to story mechanism decreases. Nakashima and Sawaizumi (1999) simplified a frame structure into a fishbone shaped model to perform dynamic analysis with earthquake motion as input and indicated that the necessary COF value that ensures beam hinging responses increases steadily with the increase of the ground motion amplitude. Medina and Krawinkler (2005) studied a family of regular frames to evaluate the strength demands relevant for the seismic design of the columns and indicated that the potential of plastic hinging in columns is high for the frames designed according to the strong column weak beam requirements of current code provisions. Kawano et al. (1998) presented a basic knowledge on the COF for forming the weak-beam type of plastic mechanisms in steel reinforced concrete frames. Dooley and Bracci (2001) investigated the influence of the COF at the joints in two RC frame structures under seismic excitation using inelastic time-history dynamic analyses.


Figure 1. Entire Beam Hinging Failure Mode

Most of these studies used deterministic approach for specific structures and the probability of the undesirable failure mode and the risk of failure of the structure remain unknown. Since large uncertainty is associated with the member strength and the earthquake load, the use of probabilistic approach enables the structural safety to be treated in a more rational way. Taking into account these uncertainties, a probabilistic evaluation method (Ono et al. 2000; Zhao et al. 2002) is applied to COF evaluation. In the present study, the base shear distribution of the Bangladesh National Building Code (BNBC) is taken into account. Based on the investigations, the least values of COF that ensure probabilistically the preferable entire beam hinging failure mode prior to story collapse are evaluated.

## 2. Basic assumptions

The column overdesign factor (COF) is defined for each beam-column node as the ratio of the sum of the moment capacity of columns to the sum of the moment capacity of beams as:
$\operatorname{COF}(k)=\sum \mu_{m c i} / \sum \mu_{m b i}$
where $k=$ the $k$ th node; $\mu_{\text {mci }}=$ the mean plastic moment strength of column connected in the $k$ th node and $\mu_{m b i}=$ the mean plastic moment strength of beam connected in the $k t$ h node.

For ductile frame structure considered in this study the following basic assumptions are used:

- Elastic-plastic frame structures are considered. The failure of a section means the imposition of a hinge and an artificial moment at that section.
- The structural uncertainties are represented by considering only the moment capacities as random variables. The coefficient of variation of material strength is considered to be 0.1 .
- All the random variables are assumed to follow the lognormal distribution. The random variables are also assumed to be statistically independent.
- The external load considered is only the lateral earthquake load. The base shear distribution of the Bangladesh National Building Code (BNBC) is taken into account. The coefficient of variation of the earthquake load is considered to be 0.8 .
- Geometrical second-order and shear effects are neglected. The effect of axial forces on the reduction of moment capacities is also neglected.
- All beam-column nodes have identical COF, i.e., there is only one value of COF for a structure.


## 3. Brief description of base shear distribution

The static equivalent base shear is defined in the Bangladesh National Building Code (BNBC), as:

$$
\begin{equation*}
V=\frac{Z I C}{R} W \tag{2}
\end{equation*}
$$

where $W$ is the total dead load, $Z$ is the seismic zone coefficient, $I$ is the importance factor, $R$ is the response modification coefficient and $C$ is numerical coefficient given by the relation:
$C=\frac{1.25 S}{T^{2 / 3}} \leq 2.75$
where $S$ is the site soil coefficient and $T$ is the fundamental time period calculated as:
$T=C_{t}\left(h_{n}\right)^{3 / 4}$
where $h_{n}$ is the height in m and $C_{t}$ is equal to 0.083 for steel moment resisting frame. The base shear will be distributed along the height according to the relation:
$F_{x}=\frac{\left(V-F_{t}\right) w_{x} h_{x}}{\sum_{i=1}^{n} w_{i} h_{i}}$
where, $F_{t}$ is the concentrated force acting at the top (roof) of the structure in addition to the $F_{x}$ force at that level. For $T$ greater than 0.7 second; $F_{t}=0.07 T V \leq 0.25 \mathrm{~V}$, otherwise it is equal to zero.

## 4. Failure modes analysis

There are too many failure modes for a frame structure. Among all the failure modes the story collapse to one or more stories is the most dangerous one. Therefore, the present study is conducted based on this type of failure mode. For convenience, the story collapse modes are defined before the probabilistic evaluation so that the investigation can be carried out in each type sequentially.

In this study, the story failure modes are classified into three patterns: lower story failure pattern, middle story failure pattern and upper story failure pattern, which depend on the location of the failure stories, as shown in Fig. 2. Lower story failure pattern is characterized by the continuous collapse of stories from the first story of the frame; upper story failure pattern is characterized by the continuous collapsed stories from the top story of the frame; in middle story failure pattern, the mechanism occurs in the middle stories of the frame and the stories at the top and bottom remain elastic.


Figure 2. Story Collapse Modes

Based on the principle of virtual work, performance function for these three story failure patterns can be established as follows:
$G_{L}(\mathbf{X})=2 \sum_{j=1}^{n_{c}-1} \sum_{i=1}^{m} M_{b i j}+\sum_{l=1}^{4} M_{c s l}+\sum_{l=1}^{2 m-2} M_{c l}-\sum_{j=1}^{n_{c}}\left(\sum_{i=1}^{j} h_{i} P_{j}\right)-\sum_{i=1}^{n_{c}} h_{i} \sum_{j=n_{c}+1}^{n} P_{j}$
$G_{M}(\mathbf{X})=2 \sum_{j=1}^{n_{c}-1} \sum_{i=1}^{m} M_{b i j}+\sum_{l=1}^{4} M_{c s l}+\sum_{l=1}^{2 m-2} M_{c l}-\sum_{j=1}^{n_{c}}\left(\sum_{i=n_{b}+1}^{n_{b}+j} h_{i} P_{j+n_{b}}\right)-\sum_{i=n_{b}+1}^{n_{b}+n_{c}} h_{i} \sum_{j=n_{c}+1}^{n-n_{b}} P_{j+n_{b}}$
$G_{U}(\mathbf{X})=2 \sum_{i=1}^{m} M_{b n i}+2 \sum_{j=n-n_{c}+1}^{n-1} \sum_{i=1}^{m} M_{b i j}+\sum_{l=1}^{2} M_{c s l}+\sum_{l=1}^{m-1} M_{c l}-\sum_{j=n-n_{c}+1}^{n}\left(\sum_{i=n-n_{c}+1}^{j} h_{i} P_{j}\right)$
where $G_{L}, G_{M}$ and $G_{U}$ are the performance functions of the lower story failure pattern, middle story failure pattern and upper story failure pattern respectively. $M_{b n i}$ is the moment strength of the beam of the top story, $M_{b i j}$ is the moment strength of the beam of the $i$ th span and $j$ th story, $M_{c l}$ is the moment strength of an interior column, $M_{c s l}$ is the moment strength of an exterior column, $P_{j}$ is the load acting on the $j$ th story of the structure, $n$ is the number of stories, $n_{c}$ is the number of failure stories, $n_{b}$ is the number of unbroken stories at the bottom of the structure, $m$ is the number of spans and $h$ is the story height of the structure.


Mode- 1 Mode- $2 \quad$ Mode- $3 \quad$ Mode- $4 \quad$ Mode-5
Figure 3. Lower Story Collapse Modes of a Six Story Frame


Figure 4. Failure Probability of Lower Story Collapse Modes

The middle collapse modes of the six storied frame are shown in Figure 5. The failure probabilities of these modes are shown in Figure 6.


Mode-1 Mode-2 Mode-3 Mode-4 Mode-5 Mode-6 Mode-7 Mode-8 Mode-9 Mode-10
Figure 5. Middle Story Collapse Modes of a Six Story Frame


Figure 6. Failure Probability of Middle Story Collapse Modes


Mode- 1 Mode- $2 \quad$ Mode- $3 \quad$ Mode- $4 \quad$ Mode- 5
Figure 7. Upper Story Collapse Modes of a Six Story Frame


Figure 8. Failure Probability of Upper Story Collapse Modes

If the number of failure stories $n_{c}$ is equal to the number of stories $n$ then the upper collapse mode will transform into the entire beam hinging failure mode shown in Fig. 1. So the performance function of the beam hinging failure mode can be obtained from Eq. 8 by putting $n_{c}=n$ as follows:
$G_{B}(\mathbf{X})=2 \sum_{i=1}^{m} M_{b n i}+2 \sum_{j=1}^{n-1} \sum_{i=1}^{m} M_{b i j}+\sum_{l=1}^{2} M_{c s l}+\sum_{l=1}^{m-1} M_{c l}-\sum_{j=1}^{n}\left(\sum_{i=1}^{j} h_{i} P_{j}\right)$
where $G_{B}$ is the performance function of the beam hinging failure mode.
To make the structure designed with same COF in all the beam column nodes, the mean values of the member strengths are assumed to have the following relationship

$$
\begin{equation*}
\mu_{b n i}=\mu_{b} ; \mu_{b i j}=2 \mu_{b} ; \mu_{c s l}=\operatorname{COF}^{*} \mu_{b} ; \mu_{c l}=2 \operatorname{COF}^{*} \mu_{b} \tag{10}
\end{equation*}
$$

where $\mu_{b}$ is the mean value of moment strength of the beam of the top story, $\mu_{b i j}$ is the mean value of moment strength of the beam of the $i$ th span and $j$ th story, $\mu_{c s l}$ is the mean value of the moment strength of an exterior column and $\mu_{c l}$ is the mean value of the is the moment strength of an interior column. The mean value of moment strength of the beam of the top story is assumed and mean value of moment strength of all other members are obtained from the above relation. For an example, let us consider a six storied two bay frame. For this six storied frame, there are five lower collapse modes. These collapse modes are shown in Fig. 3. The failure probabilities of these modes are shown in Fig. 4. It is observed that probabilistic order of the lower failure modes is greatly affected by the COF of the frame. The middle collapse modes of the six storied frame are shown in Figure 5. The failure probabilities of these modes are shown in Figure 6. It is observed that in all cases the failure probability with higher $\mathrm{n}_{\mathrm{b}}$ is less than that with lower $\mathrm{n}_{\mathrm{b}}$. The upper collapse modes of the six storied frame are shown in Figure 7.

The failure probabilities of the upper story failure modes are shown in Figure 8. It is observed that the failure probabilities of the upper story failure modes steadily increase with the increase of the number of failure stories.

For a multi-story frame the number of potential story mechanism is quite large and it increases with the increase of the number of stories. But an earlier study on the story failure modes of the frame structures (Zhao et al. 2007) showed that all the lower story collapse modes and the upper story collapse modes with highest failure stories are the most likely failure modes. So these modes are considered in COF evaluation. Therefore, in this case of the six storied frame all the lower collapse modes shown in Figure 3 and the upper story collapse modes with highest failure stories which is in this case the last mode shown in Figure 7 are the most likely failure modes.

## 5. Target COF for avoiding story mechanism

To avoid probabilistically the story mechanisms, the probabilities of the story mechanisms should be controlled at least lower than that of the entire beam hinging failure mode. In the target COF evaluation, following evaluation index is used:

$$
\begin{equation*}
\gamma=P_{f 2} / P_{f 1} \tag{11}
\end{equation*}
$$

where $P_{f 1}=$ the occurrence probability of the beam hinging failure mode and $P_{f 2}=$ the occurrence probability of the most likely story mechanism.

In the target COF evaluation, the reliability index of the entire beam hinging mode $\beta_{T}$ should be given first to indicate the safety requirement of the structure. $P_{f 1}$ is the probability corresponding to the reliability index, namely
$P_{f 1}=\Phi\left(\beta_{T}\right)$
The probabilities of the preferable collapse mode and the undesirable collapse mode should be calculated under the same load conditions; otherwise the evaluation index $\gamma$ is meaningless. The method used in this paper is to assume a reliability index for the entire beam hinging failure mode first to specify the safety level of the structure and then to compute the mean value of the earthquake load by first order reliability method FORM (Ang and Tang 1984). This load is then applied to compute the probabilities of the undesirable story mechanisms.

After obtaining the evaluation index mentioned above, to ensure probabilistically that the designed structure collapses according to the designed preferable failure mode, the relative occurrence rate of the most likely story mechanism $\gamma$ should be controlled lower than a specific allowable level $\gamma_{0}$ as follows:
$\gamma=P_{f 2} / P_{f 1} \leq \gamma_{0} \leq 1$
By conducting the failure mode analysis and the reliability analysis using different COF for a frame structure, a $\gamma$-COF curve can be obtained and the target value of the COF for which Eq. 13 is satisfied can be determined. The larger the value of COF, the smaller the value of the relative occurrence rate of the undesirable failure modes.

The target COF has been evaluated for four to six storied frame having equal bay width of 8 m and equal story height of 4 m . The $\gamma$-COF curves are shown in Fig. 9. The COF corresponding to $\gamma_{0}$ equal to one is the minimum COF value to avoid probabilistically the story mechanisms, is termed here as target or basic COF. The COF value lower than this value will enhance the story collapse i.e., the probabilities of the story mechanisms will be higher than that of the beam hinging mode. At $\gamma_{0}=1.0$ the occurrence probabilities of the story mechanisms is at least not greater than that of the beam hinging mode, rather both the probabilities are equal. The COF value higher than this value will ensure probabilistically the preferable beam hinging mode and will avoid probabilistically the undesirable story collapse modes.

The target COF for four to six storied building frames under reliability level 2,3 and 4 ( $\beta_{T}=2, \beta_{T}=3$ and $\beta_{T}=4$ ) have been presented in Table 1.

Table 1
Target COF requirement for Multi-Story Frames

|  | Four story | Five story | Six story |
| :--- | :--- | :--- | :--- |
| $\beta_{T}=2$ | 1.37 | 1.48 | 1.63 |
| $\beta_{T}=3$ | 1.19 | 1.24 | 1.33 |
| $\beta_{T}=4$ | 1.10 | 1.13 | 1.17 |



Figure. 9. $\gamma$-COF Curve for Multi-Story

It is observed that under same reliability level the target COF requirement increases with the increase of the number of story and it decreases with the increase of the reliability index.

In the present study, COF requirement of the multistory ductile frame structures has been evaluated considering the uncertainties of earthquake load and strengths of structural members based on seismic base shear distribution of Bangladesh National Building Code (BNBC). This study will guide the engineers to select the minimum values of COF for frame structures under specific reliability level to avoid probabilistically the undesirable story collapse modes during earthquakes.


Figure 10. COF with Number of Story

## 6. Conclusion

In the present study, initially the failure modes of the multistory ductile frames structure grouping into three types: upper story collapse, middle story collapse and lower story collapse are investigated probabilistically applying first order reliability method (FORM) considering seismic base shear distribution of Bangladesh National Building Code (BNBC). Then, the COF requirement that ensure probabilistically the preferable entire beam hinging failure mode and avoid probabilistically the undesirable story collapse modes of the frame structure during earthquakes has been evaluated. The findings of the paper are summarized as follows:
(1) The failure probabilities of the middle story and upper story collapse modes follow some specific pattern but the failure probabilities of the lower story collapse modes don't follow any specific pattern. It is observed that in all cases of middle story collapse modes the failure probability with higher $n_{b}$ is less than that with lower $n_{b}$. In case of upper story collapse modes, it is observed that the failure probability steadily increase with the increase of the number of failure stories.
(2) It is found that under same reliability level target COF requirement increases with the increase of the number of story and it decreases with the increase of the reliability level.

## References

Anderson J.C. and Gupta R.P. (1972), Earthquake resistant design of unbraced frames. Journal of Structural Engineering, ASCE; 98(11), 2523-2539.
Ang A.H-S. and Tang, W. (1984), Probability concepts in engineering planning and design. Vol. II-Decision, Risk and Reliability, Wiley \& Sons, New York.
ANSI/AISC (2005), Seismic Provision of Structural Steel Buildings, American Institute of Steel Construction, Washington.
ASCE (2005), Minimum design loads for building and other structures, ASCE/SEI 7-05, American Society of Civil Engineers, Reston, Virginia.
BCJ (2004), Cold-formed square tube design and construction manual, The Building Center of Japan, Tokyo, Japan.
Clough R.W. and Penzien J. (1982), Dynamics of structures. McGraw-Hill, Singapore.
Dooley K. L. and Bracci, J. M. (2001), Seismic evaluation of column-to-beam strength ratios in reinforced concrete frames. Structural Journal, ACI; 98(6), 843-851.
HBRI (1993), Bangladesh National Building Code (BNBC), Housing and Building Research Institute and Bangladesh Standards and Testing Institution, Dhaka, Bangladesh.
Hibino Y. and Ichinose T. (2005), Effects of column-to-beam strength ratio on seismic energy distribution in steel frame structures. Journal of Structural Engineering; 51B, 277-284.
ICC (2006), International building code, International Code Council, Whittier, California.
Kawano A., Matsui C., and Shimizu R. (1998), Basic properties of column over design factors for steel reinforced concrete frames. Journal of Structural and Construction Engineering; No. 505 153-159.
Lee H.S. (1996), Revised rule for concept of strong-column weak-girder design. Journal of Structural Engineering, ASCE; 122(4), 359-364.
Medina RA. and Krawinkler H. (2005), Strength demand issues relevant for the seismic design of moment resisting frames. Earthquake Spectra, 21(2), 415-439.
MLIT (2008), The building standard law of Japan, The Ministry of Land, Infrastructure and Transport, Tokyo, Japan.
Nakashima M. and Sawaizumi, S. (1999), Effects of column-to-beam strength ratio on earthquake response of steel moment frame (Part 1 : column-to-beam strength ratio required for ensuring beam-hinging responses). Steel Construction Engineering; 6(23), 117-132.
Ono T., Zhao Y. G. and Ito T. (2000), Probabilistic evaluation of column over design factor for frames. Journal of Structural Engineering, ASCE; 126(5), 605-611.
Park R. and Pauly T. (1975), Reinforced concrete structures. Wiley \& Sons, New York.
Zhao Y. G., Ono T., Ishii K. and Yoshihara K. (2002), An investigation on the column over design factors for steel framed structures. Journal of Structural and Construction Engineering; No.558, 61-67.
Zhao Y. G., Pu W.C. and Ono T. (2007), Likely story mechanisms of steel frame structures. Journal of Structural and Construction Engineering; No.613, 35-42.

