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Prediction of sequent depth ratio of hydraulic jump in sloping channel with sudden drop

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Abstract

Laboratory flume experimentation has been conducted to evaluate the characteristics of hydraulic jump in a sloping channel with abrupt drop. The basic equation is based on the application of the one dimensional momentum equation and continuity equation. The format is similar to the well-known Belanger equation for classical jump with modification of Froude number. The modified Froude number term contains three additional parameters, two of them incorporate the effect of Froude number and channel slope and the third one represents for describing the effect of drop height. The results of the experimental study were used to assess the prediction model for computing sequent depth ratio in a sloping channel with abrupt drop. A recalculating tilting flume has been used to carry out the experimental investigations. A total of 108 test runs have been conducted for varying slope, drop height and flow. The initial depth, sequent depth, velocity etc. were measured with different combinations of drop height and channel slope. The unknown parameters of basic equation were calculated from derived equations and evaluated with the experimental data to relate with inflow Froude number. Thus a simpler approach has been suggested to predict the sequent depth ratio of hydraulic jump in a slopping channel with abrupt drop.

Keywords: Hydraulic jump, sequent depth ratio, laboratory flume, sudden drop, sloping channel.

1. Introduction

Many hydraulic works (i.e. urban drainage networks, river channelizations, spillways, irrigation channels) require a flow transition from supercritical to subcritical, the relative hydraulic jump being steadily located within a certain short channel stretch (stilling basin). Without suitable measures, the hydraulic jump would be located outside the stretch itself. If the momentum of the supercritical flow exceeds that of the subcritical one, the steady location of the hydraulic jump within the stilling basin is obtained by means of a drop or enlargement of the channel width i.e. the section of the downstream channel is increased. In many situations, each of the above measures (either bottom rise or width reduction, either drop or enlargement) can by itself prove able to steadily locate the jump within the stilling basin. However, specific hydraulic building situations could require a combination of bottom rise and width reduction or drop and enlargement. (Ferreri & Nasello 2002). Hydraulic jumps in abrupt drop may be described by the approaching conditions, that is the inflow depth y_1, the inflow Froude number F_1 and the position of toe x relative to the expansion section.

In this study, a hydraulic jump in the section of abrupt drop of a sloping channel is considered. In practice one should anticipate a situation when the tail water depth is not equal to the required conjugate depth at all discharges. The jump can then form close to the structure, be drowned or be repelled downstream at different discharges. Ideally the jump should form close to the structure and certain appurtenances or artifices are used to control the location of the jump, i.e. to force the jump to occur at a desired location. The jump is then known as a forced jump. The devices used for the purpose may be baffle blocks and sills or a depression or rise in the floor level. Jump at an abrupt drop, jump at an abrupt rise, jump under influence of cross jets are some examples of this case. If jumps are required to force in a certain location sometimes abrupt drop of the channels can be useful when there is no space for the sudden expansion of the channel section. Again, this situation may occur in a sloping channel like weirs with sloping faces or spillways. In such basins, there are mainly two problems faced by the field engineers who monitor the performance of the design. One is the determination of sequent depth and the other is the estimation of energy loss.

Matin et al. (1997, 2008) developed a Belanger's format prediction model for computing sequent depth ratio of hydraulic jumps in abruptly expanding channel based on the application of the one-dimensional momentum and continuity equations. Evaluation of the coefficients and exponents of the developed equation need experimental data for its use in practice. The validity range of the parameters used in the developed equation has also known from the experimental study. This model contains three unknown parameters and experimentation is required to evaluate these parameters. Form experimental data it will be possible to develop a mathematical relationship in terms of some known variables incorporating height of drop, channel slope, upstream Froude number etc. Therefore, the present study is directed towards the evaluation of related parameters using the experimental data in the laboratory flume to develop prediction model to determine the sequent depth in sloping channel with sudden drop.

3. Review of previous investigations

Many experimental investigations were carried out to compute the sequent depth ratio and other related parameters of hydraulic jump in a sloping channel. Some prominent works in this topic of recent times were carried out by Hager (1989, 1992), Ohtsu and Yasuda (1991), Husain et al. 1995. Hydraulic jump in a horizontal channel at an abrupt drop had been reported by Rajaratnam and Ortiz 1977, Ranga Raju 1993, Caisley 1999. They found that for the wave form of the hydraulic jump at an abrupt drop, the upstream supercritical flow jet is deflected upwards into a wave formation as a result of back pressure below the drop. Then the jet plunges into the tailwater and strikes the downstream bed of the river**.** The authors also noted that the formation of a wave can be completely eliminated by a rounded step, which allows the supercritical flow jet to deflect downwards at the drop. It was found that hydraulic jumps at abrupt drops have certain oscillatory characteristics (Mossa 1999 and Mossa et al. 2003).

It is revealed from the above review that many researchers have been carried out study to analyze the hydraulic jump in horizontal prismatic channels with abrupt drop. No notable works have been carried out to compute the sequent depth ratio for hydraulic jump in a sloping channel with abrupt drop.

4. Theoretical formulation

The definition sketch for a hydraulic jump in a sloping channel with abrupt drop is shown in Figure 1. The toe of the hydraulic jump is located at the drop section.

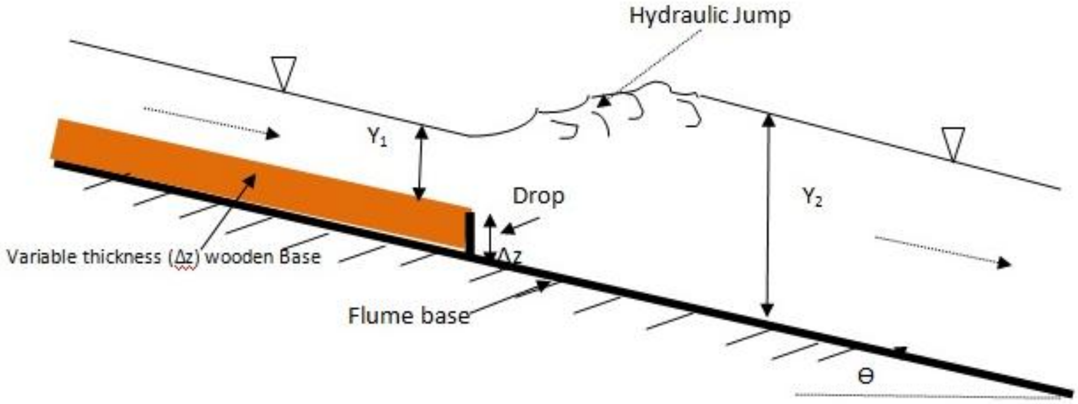


Fig. 1. Definition sketch of a hydraulic jump in a sloping channel with abrupt drop.

Applying the both momentum, continuity equation, and introducing non dimensional terms D for sequent depth ratio = y_2/y_1 and F_1 for inflow Froude number = $\frac{U_1}{\sqrt{g y_1 \cos\theta}}$, the following expression can be derived:

$$(D^2+D)\left[\cos\theta - \frac{KL_j \tan\theta}{y_2-y_1} + \left(2y_1\Delta z + \frac{\Delta z^2}{\cos^2\theta}\right)\frac{1}{y_1^2-y_2^2}\right] = 2F_1^2 \quad (1)$$

To get a solution for D in the Belanger's format, equation 11 is modified as:

$$D^2 + D - 2G_1^2 = 0 \quad (2)$$

The sequent depth ratio D is obtained by solving equation (2) as

$$D = \frac{1}{2}\left(\sqrt{1+8G_1^2} - 1\right) \quad (3)$$

Where, G_1 = modified Froude number

The relationship between G_1 and F_1 can be rearranged as

$$G_1^2 = \frac{F_1^2}{k_1\left\{(1-k_2)\frac{\cos\theta}{k_1} - k_3\right\}} \quad (4)$$

Where k_1, k_2 and k_3 can be defined as,

$$k_1 = \frac{1}{y_2^2 - y_1^2} \quad (5)$$

$$k_2 = \frac{KL_j \tan\theta}{(y_2+y_1)^2(y_2-y_1)\cos\theta} \quad (6)$$

$$k_3 = 2y_1\Delta z + \frac{\Delta z^2}{\cos\theta} \quad (7)$$

It is obvious that establishing the relation of sequent depth ratio requires determination of three factors k_1, k_2 and k_3.

Where k_1 is a function of upstream depth and downstream depth of hydraulic jump. k_2 is a function of dimensionless jump length and modifying factor K. k_3 is a function of upstream depth and depth of the sudden drop. k_2 and k_3 are again a function of θ. It is possible to find out factors k_1, k_2 and k_3 from the experimental data and then the sequent depth ratio can be found from the Equation 3.

5. Flume experiment

The transition from supercritical to subcritical flow in a sloping channel with abrupt drop in bed is systematically investigated under a range of slopes. The experimental study was conducted by Sultana 2011. At the Hydraulics and River Engineering Laboratory of the Department of Water Resources Engineering of Bangladesh University of Engineering and Technology. The experimental setup involved the use of a laboratory tilting flume having an adjustable sluice gate and an adjustable tailwater gate, water tank, pump, water meter and various constriction elements. The experiments were performed in the 40-ft long tilting flume in the laboratory. Tilting facility of the flume was used to make it to a sloping channel. It was possible to create only mild slopes in this artificial channel (highest possible slope is 1 in 40). Three different slopes of 0.0042, 0.0083 and 0.0125 were maintained in the flume. To create a hydraulic jump in the channel it is necessary to install a sluice gate in the channel.

Plate 1: Photograph of the 12.2 m (40-ft) long Laboratory tilting flume

Plate 2: a) Side view b) Top view

Fig. 1. Close views Hydraulic jump at sudden drop is a sloping channel (top view)

For maintaining abrupt drops Δz, several constriction elements were installed in the stilling chamber in the laboratory flume. They were made of well-polished wood. A constriction element of height 2 cm and length of 426.72cm (14 ft) was installed in the bed of the flume. There was no lateral movement of water between the constriction elements and the sidewalls because of water tightness of these elements. A series of experiments were performed with a step height of 2 cm, 4.5 cm and 6 cm. These heights were obtained by changing thickness of the constriction element in the channel bed. The channel width is 0.3048m (1-ft) and the sidewall height is approximately 0.3048 m (1-ft). It is supported on an elevated steel truss that spans the main supports (Plate 1). The channel slope can be adjusted using a geared lifting mechanism.

The whole flume consists of an upstream reservoir and a stilling chamber with contraction reach. The original channel depth was reduced by various constrictions. All constriction elements were made of wood that was located in the bed of the channel. The flume has an adjustable sluice gate and an adjustable tailwater gate located, respectively, upstream and downstream of the expansion geometry. The tail water depth was controlled by a vertical gate located at the downstream end of the flume. Water issuing through an opening of the sluice gate, located downstream from the reservoir, formed the supercritical stream. During the experiments, the location of the hydraulic jump was controlled by the downstream gate and discharge. The sluice gate and the flow discharge control the quasi-uniform flow upstream of the jump and the tailwater gate acts as a downstream control.

The circulation of the water within the flume is a closed system. From the storage reservoir the water is transported by means of the pipeline to the upstream reservoir. There are two types of pipelines via suction and delivery pipeline. Suction pipe sucks the water from the storage reservoir and at the same time passes that water through the pump. The water is delivered to the channel through the delivery pipe and returns to the storage reservoir.

Table 1
Summary of experimental data range for different flow and geometric conditions

No. of Runs	Drop, Δz (m)	Slope	Flow rate Q (m^3/s)	y_1 (m)	y_2 (m)	Sequent depth ratio (D)	U (m/s)	Froude No. ($F_{1)}$
9	0.02	0.000	0.013-0.0215	0.017-0.035	0.111-0.149	3.68-6.94	0.75-1.26	1.35-2.65
9	0.02	0.0042	0.012-0.0226	0.022-0.033	0.128-0.169	4.06-6.46	0.61-1.24	1.17-2.37
9	0.02	0.0083	0.013-0.0226	0.022-0.033	0.129-0.183	4.19-7.13	0.80-1.50	1.09-2.57
9	0.02	0.0125	0.013-0.0226	0.022-0.035	0.140-0.184	4.80-6.83	0.81-1.56	1.42-3.22
9	0.045	0.000	0.012-0.0250	0.020-0.028	0.150-0.190	5.36-9.00	0.90-1.80	1.72-3.79
9	0.045	0.0042	0.0139-0.215	0.020-0.038	0.135-0.195	5.36-9.00	0.81-2.00	1.59-3.28
9	0.045	0.0083	0.012-0.0215	0.020-0.046	0.140-0.189	5.36-9.00	0.70-2.92	1.58-3.82
9	0.045	0.0125	0.0157-0.020	0.021-0.400	0.135-0.201	5.54-9.00	0.80-2.13	1.55-3.67
9	0.06	0.000	0.0065-0.022	0.016-0.035	0.120-0.204	5.54-9.12	0.96-1.31	1.66-3.24
9	0.06	0.0042	0.007-0.0202	0.019-0.037	0.148-0.205	5.41-9.48	0.85-1.40	1.41-3.24
9	0.06	0.0083	0.008-0.0208	0.021-0.035	0.155-0.211	6.41-8.76	0.87-1.45	1.36-3.16
9	0.06	0.0125	0.009-0.0221	0.019-0.036	0.165-0.206	5.53- 9.95	0.83-1.45	1.42-3.27

6. Results and discussion

Experimental investigations were conducted under different flow conditions. Total of 108 test runs were conducted, the data ranges for various flow and geometric conditions are summarized Table 1. Three different drop heights (Δz = 2 cm, 4.5 cm, and 6 cm); three slopes of the channel (0.0042, 0.0083 and 0.0125) and three different gate openings (3.6 cm, 5 cm and 6.5 cm) were used. The discharges were varied accordingly with different slopes and gate openings to get the required range of inflow Froude number. Initial depth, sequent depth, discharge, average velocity and jump length were measured for the analyses. Results of different analyses are shown.

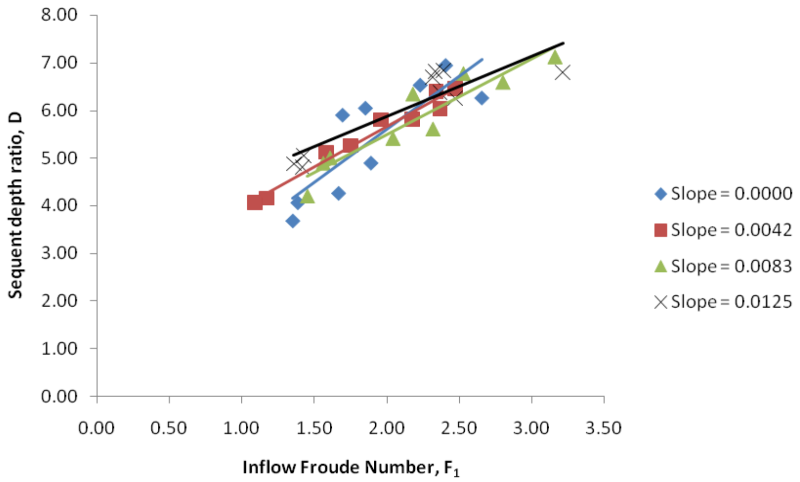


Fig. 2. D vs. F_1 for different channel slopes with drop height, $\Delta z = 2$ cm.

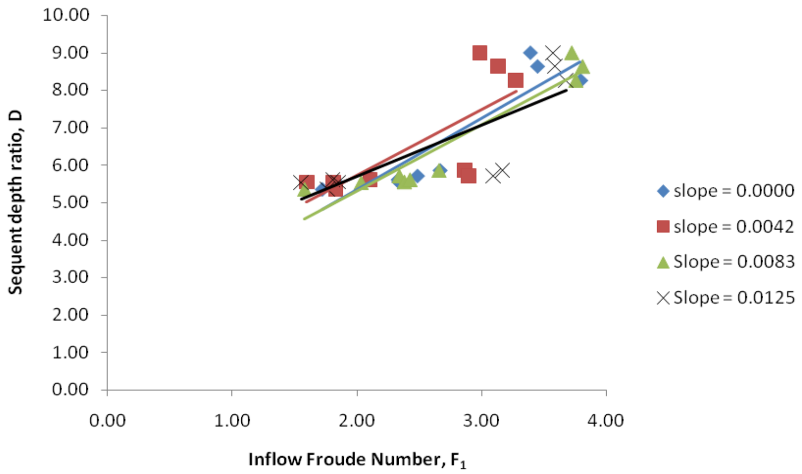


Fig. 3. D vs. F_1 for different channel slopes with drop height, $\Delta z = 4.5$ cm

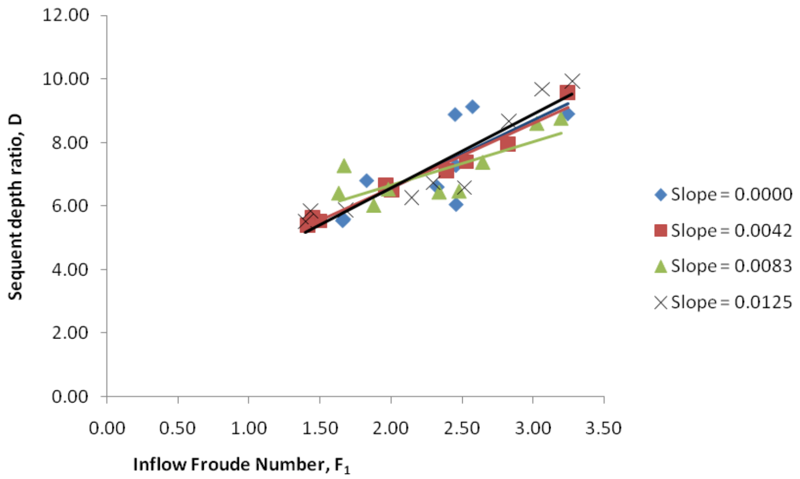


Fig. 4. D vs F_1 graph for different channel slopes with drop height, $\Delta z = 6$ cm

Evaluation of parameters k_1 k_2 and k_3: Theoretically the parameter k_1 is dependent on upstream depth, y_1 and downstream depth, y_2 and sequent depth ratio D, the value of k_1 is calculated using Equation 5 The parameter k_1 is calculated from the set of experimental data and plotted against Inflow Froude number F_1 for different hydraulic conditions. Figure 5 shows the variation of k_1 versus Fr_1 for different drop height (Δz). It is evident from the figure that the parameter value decreases with increase in value of drop height. The channel slope has very minor influence on the magnitude of the parameter k_1.

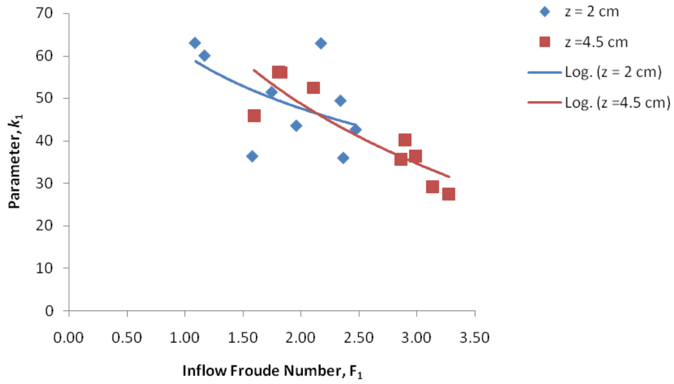


Fig. 5. Variation of parameter k_1 with F_1 for different drop height with Slope = 0.0042

The relationship representing k_1 vs. F_1 can be obtained as best fit equation as follows:

$$k_1 = -13.3 \ln (F_1) + 50.83 \quad (8)$$

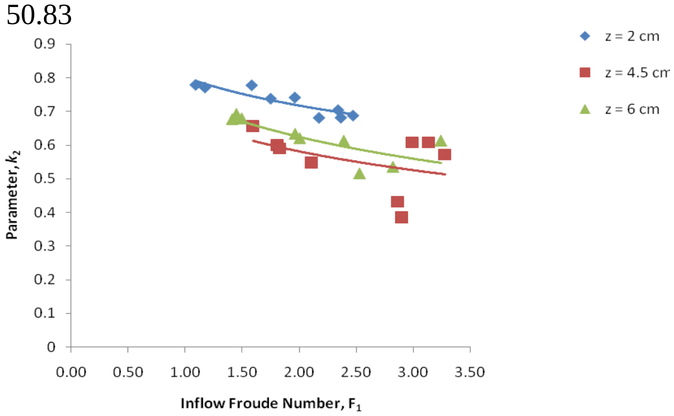


Fig. 6. Variation of parameter k_2 with F_1 for different drop height with Slope = 0.0042

The parameter k_2 has been plotted against the Inflow Froude number as shown in Figure 6 (for S=0.0042). Similar to Equation 8, the following regression equation for all the experimental data can be obtained:

$$k_2 = 0.042F_1^2 - 0.275F_1 + 1.037\cos\Theta \quad (9)$$

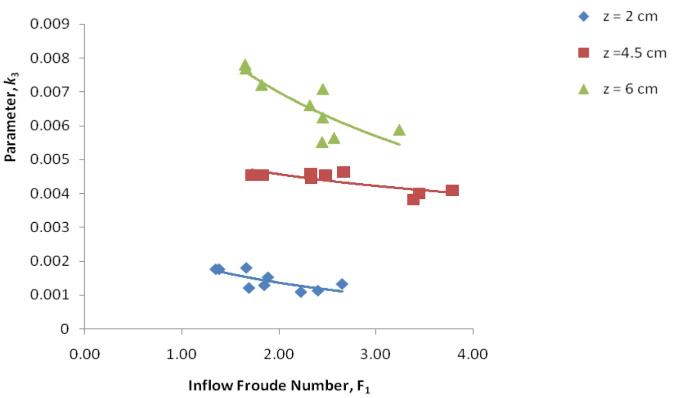


Fig. 7. Variation of parameter k_3 with F_1 for different drop height with Slope = 0.0042

Theoretically the parameter k_3 is dependent on upstream depth, y_1 and drop height, Δz and channel slope, θ. the value of k_3 is calculated from Equation 7. The parameter k_3 is calculated from the set of experimental data and plotted against Inflow Froude number F_1. A typical plot for S=0.0042 is shown in Figure 7 for different geometric conditions. It is evident from the figure that the parameter value decreases with increase of drop height. The channel slope value has very minor influence on the value of the parameter.

Prediction equation (Equation 3) developed has been compared with the observed data from the series of experiments. Sequent depth ratio versus Inflow Froude number graphs for different drop height and channel slopes are plotted with the prediction equation and with the experimental data. These are shown in from Figure 10 to Figure 13. From these figures, the performance of the prediction equation can be considered satisfactory. It reveals that for drop height, Δz = 4.5 cm, predicted D slightly differs from the observed D, for Δz = 6 cm, the equation slightly overestimates the sequent depth ratio. This may be the cause of slight mismatch of result with the experimental data. For drop height, Δz = 2 cm predicted value of D matches with reasonable satisfactory with the observed values.

(a)

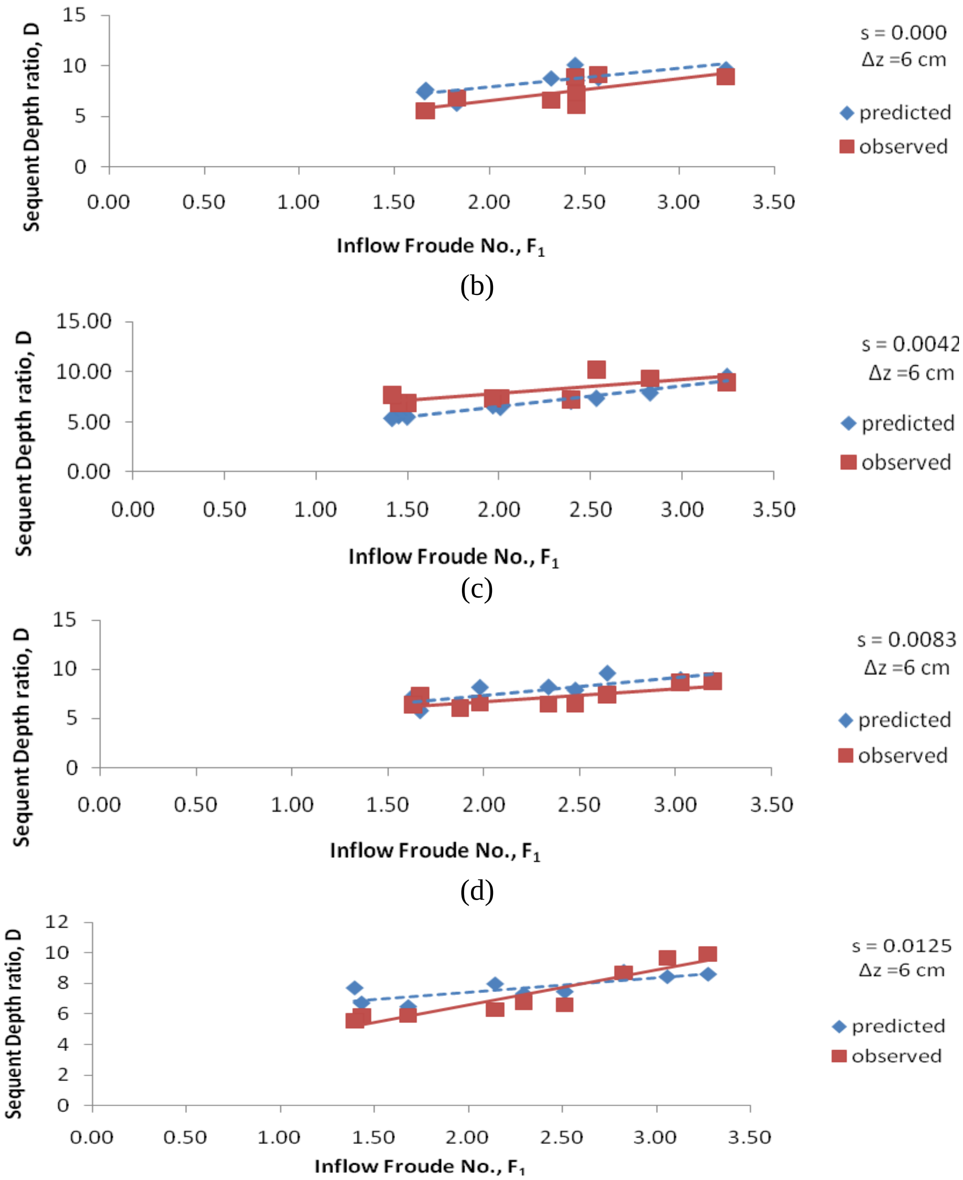
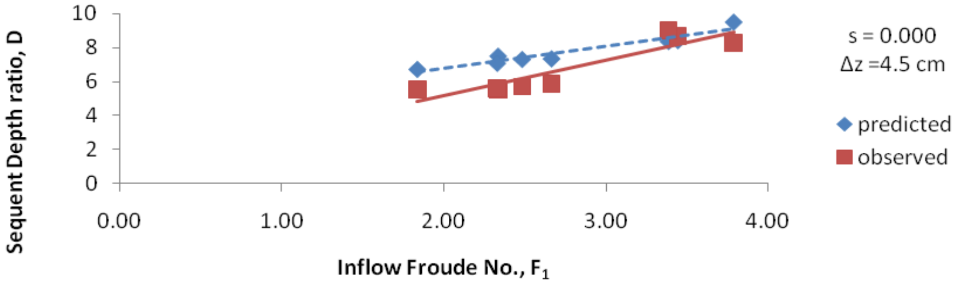
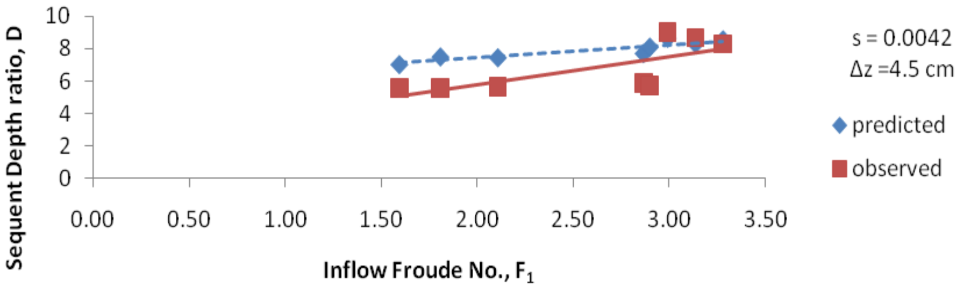


Fig. 8. D Vs F_1 with drop height, Δz = 6 cm;
(a) Slope = 0.000 (b) Slope = 0.0042 (c) Slope = 0.0083 (d) Slope= 0.0125

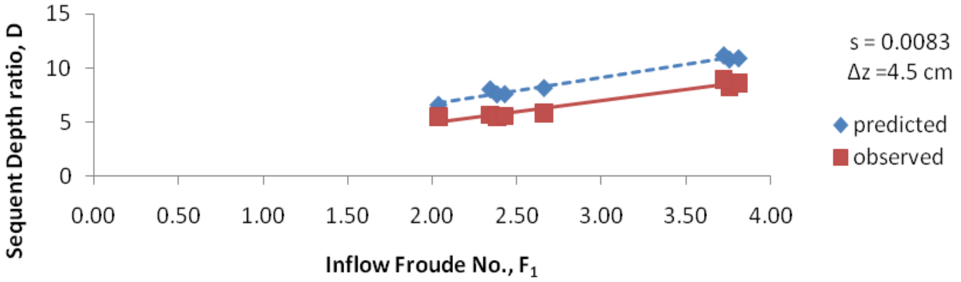
(a)



(b)



(c)



(d)

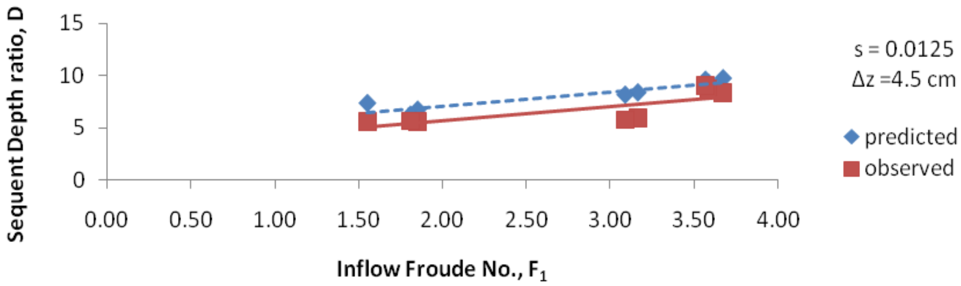
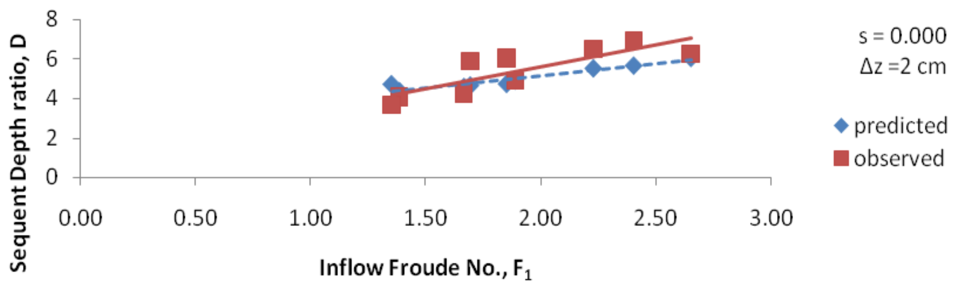


Fig. 9. D Vs F_1 with drop height, Δz = 4.5 cm;
(a) Slope = 0.000 (b) Slope = 0.0042 (c) Slope = 0.0083 (d) Slope= 0.0125

(a)



(b)

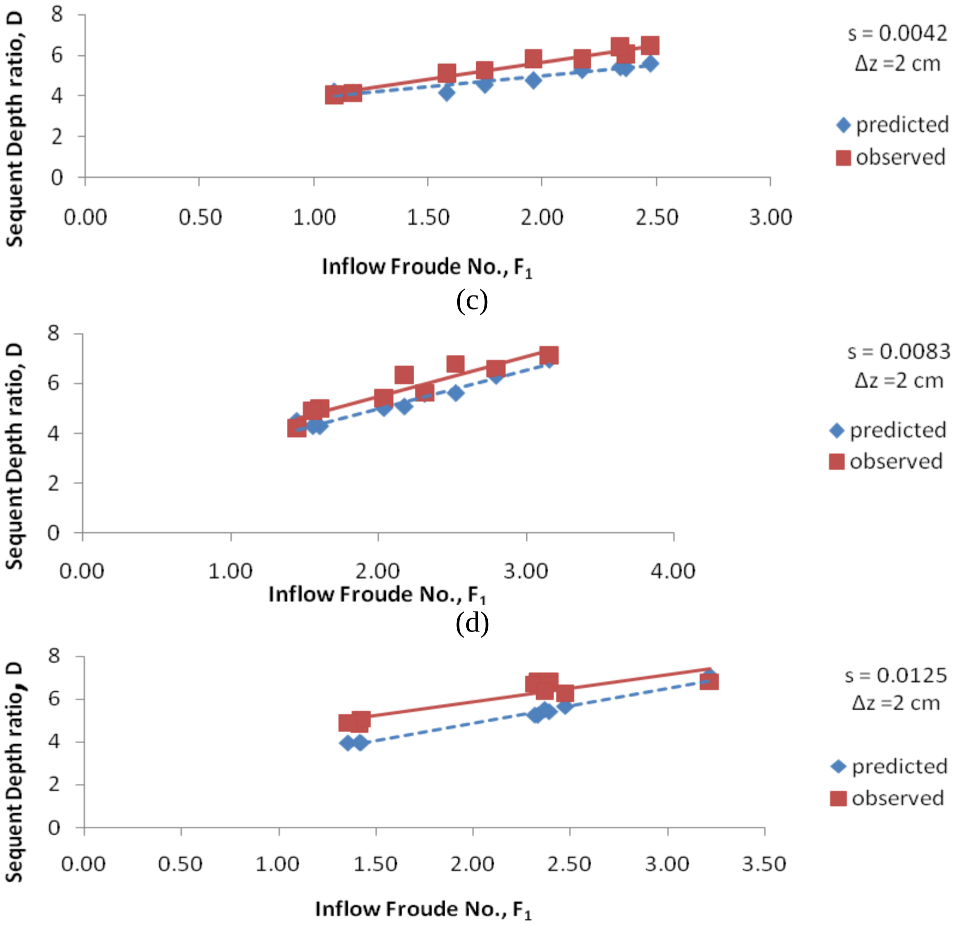


Fig. 10. D vs. F_1 with drop height, $\Delta z = 2$ cm;
(a) Slope = 0.000 (b) Slope = 0.0042 (c) Slope = 0.0083 (d) Slope= 0.0125

7. Conclusion

The theoretical equation developed for predicting the sequent depth ratio of a free hydraulic jump in a sloping stilling basin with abrupt drop. Laboratory flume experimental conditions, such as various combinations of drop height and channel slope have been conducted to evaluate the unknown parameters. Use of prediction equation in a sloping channel with abrupt drop needs three parameter k_1, k_2 and k_3 to modify the inflow Froude number. These parameters were evaluated from the experimental data and have been expressed as explicitly as function of Froude number and channel slope. Overall, a simplified approach for estimating the sequent depth of hydraulic jumps in sloping channel with abrupt drop has been suggested. It predicts the sequent depth ratio D for given drop with reasonable accuracy when compared with the experimental observed data. It can be argued that, there remains the scope of further improvement of the prediction model using experimental data for wide range of flow and channel geometric condition.

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