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A NEW DESIGN BASIS FOR FREE-STANDING STAIRS

Khan M. Amanat[1], Sohrabuddin Ahmad[1]

ABSTRACT: Current practice of analyzing free-standing stair slabs is to use approximate analytical methods due to the absence of specific code provisions. Because of their inherent limitations, these approaches cannot predict the actual 3D behaviour of the stair slab system. Analytical methods cannot predict the distribution of any stress resultants across any section. Such drawbacks of the analytical approaches manifest the necessity of the development of a more rational but simple analysis and design method. With this objective, extensive numerical study on the behaviour of the free-standing stair was made using thick shell finite elements. Sensitivity analysis of different geometric parameters and material properties has been made. The study revealed that the variation of stress resultants across a section is non-uniform, which is otherwise not recognized by the analytical methods. The sensitivity study enabled to formulate simple analytical representation of the influence of different parameters on design forces and moments. Semi-empirical equations have been proposed from which the design forces and moments can be calculated in a single step. The accuracy of the equations within acceptable limit is established through comparison with the results of rigorous FE analyses. A rational reinforcement layout scheme is also proposed recognizing the non-uniform stress distribution across stair sections. Finally, a practical design example is given which shows that the proposed design equations combined with the suggested reinforcement layout scheme leads to a simple, straight forward but rational and safe design of free standing stair slabs recognizing its true three dimensional behaviour.

KEY WORDS: Concrete, stair, free standing, analysis, design.

INTRODUCTION

Stairs are essential features of all residential and commercial buildings. From architectural point of view, free-standing stairs are more attractive than ordinary ones. However, due to the lack of a simple rational design code, designers are forced to make a conservative design resulting in an unnecessarily heavy looking structure. Although there are code provisions for ordinary dog-legged type stairs based of rigorous analytical studies (Ahmed et.al. 1995, 1996), the leading codes of practice e.g. ACI or British Code does not provide any guideline regarding the analysis and reinforcement design of this type of concrete structures.

Available simplified analytical approaches can be categorized into two types. The first type idealizes the stair slab structure as a space frame. The methods of Fuchssteiner(1954), Sauter (1964), Cusens and Kuang (1965), Taleb (1964) and Gould (1963) fall into this category. One limitation of such idealization is that these fail to predict the variation of stress

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[1] Department of Civil Engineering, BUET, Dhaka-1000, Bangladesh.

resultants across any section of the flights or landing. In the second type, the space plate configuration of the stair slab is retained but it is made determinate based on some assumptions. The overall structural rigidity resulting from the indeterminacy is lost when such assumptions are made. The methods of Sieve (1962) and Liebenberg (1960) fall into this category.

Smith (1980) discussed the behavior of a 90° free-standing stair using finite element analysis. He showed analyses using both space frame idealization and plate element idealization. Ng and Chetty (1975) analyzed a three flight free-standing stairway using both space frame idealization and determinate space plate idealization. However, none of the approaches is readily suitable for practical design because of considerable calculations. Hence there ought to be a scope for further improvement in the analysis and design procedures of free-standing stairs based on rigorous finite element analysis.

In this paper an extensive finite element investigation of a numerically modeled free-standing stairway built monolithically and supported at the floor levels is carried out. The same stairway is also analyzed using Cusens and Kuang (1965), Sieve (1962) and Sauter's (1964) approaches. The results are then compared. The influence of various material and geometric parameters on the design forces and moments are studied. Based on the findings of the study a guideline for a direct analysis and reinforcement layout of the stairway is developed.

NUMERICAL MODELING

Geometry of the stair: The stairway consists of two flights and a landing. The flights have same dimensions and are held fixed at floor levels. The *front, back, left* and *right* sides of the stair, the *inside* and *outside* of it are arbitrarily defined and are shown in Fig.1(b). The various dimensions of the prototype stairway [Fig.1 (a) and (b)] are A=305 mm, B=1220 mm, C=1220 mm, L=2550 mm and H=3050 mm. The thicknesses of flight slabs and the landing slab are the same, i.e. T_1=T_2=125 mm. Although the stairway is a concrete structure, the material is assumed to be linearly elastic, homogeneous and isotropic. The Poisson's ratio is assumed as 0.15 and the modulus of elasticity of concrete is calculated from the ACI formula, E_c= $4700\sqrt{f_c'}$ MPa, (f_c' is concrete ultimate strength in MPa). Both live load and dead loads are applied as gravity loads. A live load of $0.48\text{x}10^{-2}$ MPa is assumed. The unit weight of the material is $2.356\text{x}10^{-5}$ N/mm^3. Additional weight due to the steps is also considered. Three combinations of loading are considered depending on the position of the live load which are, (1) live load over the whole stairway, (2) live load on flights only and (3) live load on landing slab only.

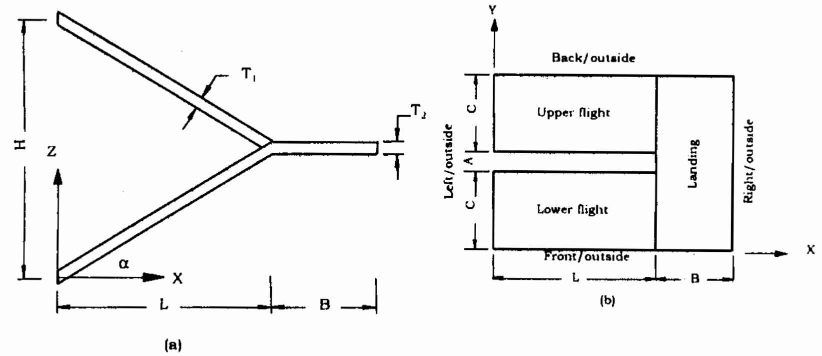


Fig 1. Stair Slab Geometry, (a) Elevation, (b) Plan

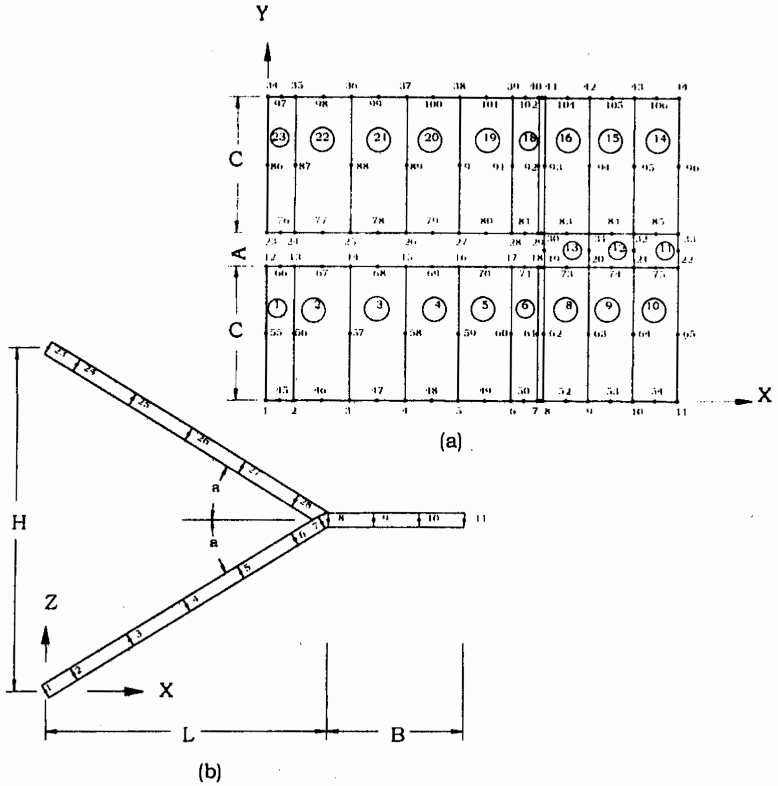


Fig 2. Finite element mesh of the stairway (a) Plan, (b) Elevation

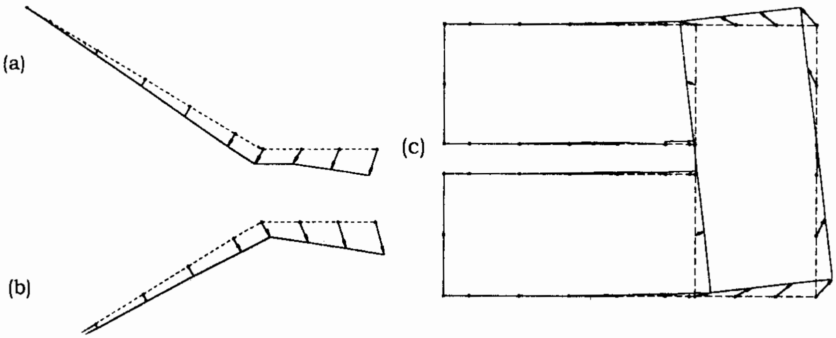


Fig 3. Deflected shape of the stairway,(a) Vertical deflection along the center of upper flight and landing, (b) Vertical deflection along the center of lower flight and landing,(c) Lateral deflection

Finite Element Modeling: The general thick shell finite element and the computer code developed by Ahmad (1969) is used in modeling the stair. A mesh comprised of 23 elements and 106 nodes was found to be adequate. The mesh is shown in Fig.2 (a) and (b).

Analytical Modeling: The same stair was analyzed using three analytical methods, namely, Cusens and Kuang's (1965) method, Siev's (1962) method and Sauter's (1964) method. The results of the analytical methods are compared with those obtained from finite element modeling. Analyses were made for all the three load cases.

FINITE ELEMENT ANALYSIS

The deflected shape of the stairway is shown in Fig.3 (a) and (b). Some typical results of finite element analysis are shown in Fig.4 (a) through (d). Figure 4(a) reveals that total bending moment at support and at mid-span of flight is maximum for load case 2 while the kink moment is maximum for load cases 1 or 3. According to Fig. 4(b), torsional moment of flight is maximum at load case 1. Figure 4(c) shows the plot of plate stress resultant (moment/unit width) across the section at support of upper flight. Figure 4(d) shows the distribution of the same for landing across the mid-landing section. It is observed from these figures that stress resultants are not uniformly distributed across the section. This clearly demonstrates that free standing stair slab, which is basically a three-dimensional plate structure, cannot be simplified to skeletal frame structure or to a determinate slab system.

It is found from the study that except the section of flight at midspan, the bending moment at other critical sections are not distributed uniformly across the section. From the observation of stresses at these locations while carrying out the parametric study it is found that at the support sections, 65 percent to 75 percent of the total bending moment occur within the outer half of the width of that section, the average being 70 percent. The inner half of the section, on an average, carries the rest 30 percent of total bending. In contrast, the inner points at flight landing junction are stressed higher than the outer points. On an average, the inner half of the section carries 63 percent of the total kink moment and outer half takes care of the rest. At the mid-landing section the variation is somewhat rapid for the inner one-third of the section [see Fig. 4(d)]. The bending stresses do not vary to any significant amount across the rest of the section. On an average, 50 percent of the total bending at mid-landing section is resisted by the inner one-third of the section and the outer two-third carries the rest.

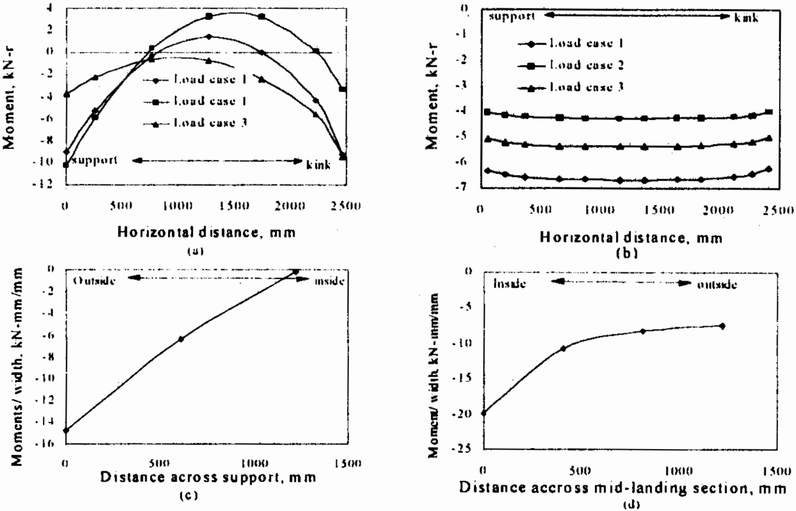
COMPARATIVE STUDY

A comparative study of the analytical methods and the finite element analysis was made to assess the relative merits and demerits of each method. Figure 5(a) shows the bending moment diagram of flights obtained by various methods and a reasonable agreement is seen between finite element analysis and other analytical methods. However, one limitation of the analytical methods is that they can give only the total moment at a particular section and cannot give the lateral distribution of the total moment across that section [see Fig.4 (c) and (d)]. Lateral distribution is important in designing the reinforcement layout. Figure 5(b) shows the bending moment in landing where it is seen that the analytical methods underestimate the moment by a substantial amount.

SENSITIVITY ANALYSIS

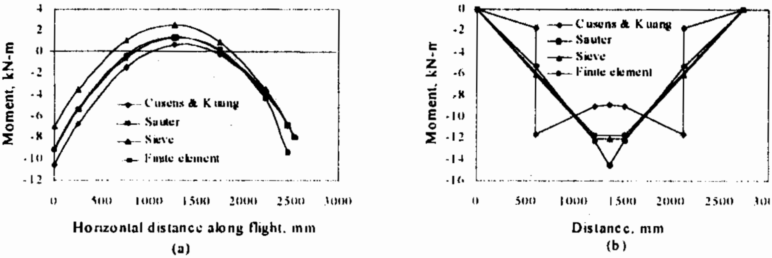
In order to develop a straightforward method of determining design forces and moments, a sensitivity analysis was done. It is noted that although analytical methods or finite element analysis enables the designers to determine forces and moments at any section of the stairway, only a few of these quantities at some critical locations are necessary for designing the stairway. These design forces and moments are, (a) Axial tension in the upper flight, (b) lateral shear at mid-span of landing, (c) torsion on flights, (d) bending moment at support, (e) bending moment at mid-span of flights, (f) bending moment at flight-landing junction, (g) bending moment at mid-span of landing, and (h) in-plane moment in flights.

The sensitivity of these quantities to the various geometric parameters are studied. Firstly, for the purpose of analysis a *datum* stairway with initial values of A = 150 mm, B = 915 mm, C = 915 mm, L = 2030 mm, H = 2440 mm and T_1=T_2=100 mm is set. Then each of these geometric parameters is varied in turn. That is, for example, when A is varied other parameters are kept at their *datum* value. The change in the magnitude of the design forces and moments with the variation of geometric parameters listed above are studied.



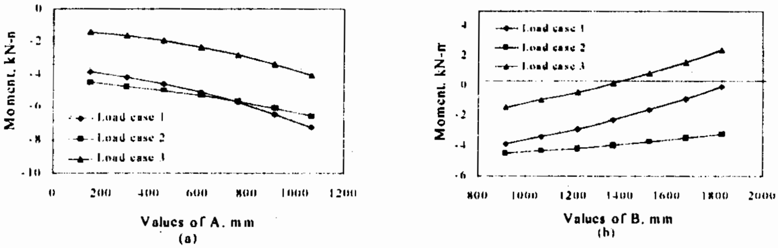
(a) (b) (c) (d)

Fig 4. Typical results of finite element analysis (a)Bending moments in flights. (b)Torsion in flights (c) Bending moment distribution across support section (d)Bending moment distribution across mid-landing section.



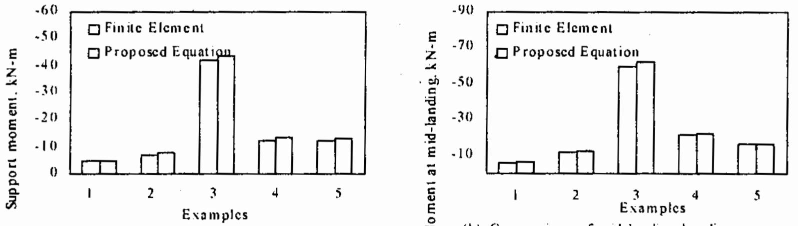
(a) (b)

Fig 5. Comparative study of analytical methods and finite element analysis (a) Comparison of flight bending moments, (b)Comparison of bending moment in landing



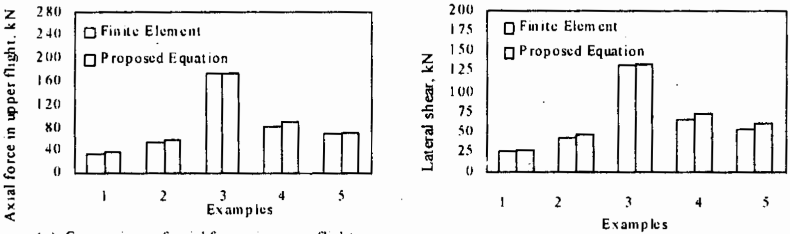
(a) (b)

Fig 6. Sensitivity analysis of support bending moment. (a) Effect of horizontal gap between flights. (b) Effect of landing width



(a) Comparison of support moments

(b) Comparison of mid-landing bending moment



(c) Comparison of axial forces in upper flight

(d) Comparison of lateral shear at mid-landing

Fig 7. Comparison of proposed equations with finite element analysis.
(a) Support moment,(b) Mid-landing bending moment, (c) Axial force,
(d) Lateral shear at mid-landing.

Due to the space limitation, only the results corresponding to the support bending moment are shown here in Fig.6 (a) and (b) for the parameters *A*, and *B*. From Fig. 6(a) it is observed that support bending moment (negative) increases with the increase of *A* while it decreases with the increase of *B* [Fig. 6(b)]. The effect of *B* can be explained by the fact that the landing act like a cantelever if we consider the kink as a support line as assumed in the analysis of Siev's method. Thus increase of *B* increases the negative moment at kink and the carry over of this moment to the support reduces the support moment.

A NEW DESIGN RATIONALE

Basis: Based on the comparative study presented above it can be said that the prediction of bending moments at different critical locations by approximate analytical methods may not always be acceptable. For example, support-bending moment is overestimated by Cusens and Kuang's approach and underestimated by Siev's approach [Fig. 5(a)] while all the methods underestimate the bending moment at landing [Fig. 5(b)]. With the advancement of the techniques of structural analysis it is now possible to analyze virtually all types of structure employing the finite element technique. However, a straightforward method of analysis to carry out the calculation more easily but with acceptable accuracy is preferable. Now, if it is possible to formulate explicit expressions for a rational and safe estimation of moments and forces at various critical locations within the acceptable limits of accuracy, it will greatly reduce the effort necessary in calculations and will speed up the design process.

Based on the detailed sensitivity analysis, approximate expressions for forces and moments at critical locations are proposed in terms of the various dimensions of the stairway which can completely define the geometry of a free standing stair. These expressions are valid within the usual ranges of the geometric parameters and concrete strength.

Equations for Forces and Moments: The expressions are explicit and empirical in nature. In these equations, the unit of force is Newton (N) and the unit of length is millimeter (mm). One limitation of the proposed guide is that the thickness of the flight and landing slabs are assumed to be equal which is most common in free-standing stairways. The equations may not always satisfy equilibrium. This is because different design parameters become critical at different loading conditions. The equations give values of moments and forces corresponding to 0.48×10^{-2} MPa live load and appropriate dead load of slab and steps assuming a unit weight of 2.356×10^{-5} N/mm^3. Since elastic analysis is made throughout, it is possible to calculate forces and moments for other values of live load by simple proportioning. With the above limitations and assumptions the equations are presented below. All of the equations are of the form,

$$\text{Force or Moment} = K.F_A.F_B.F_C.F_L.F_H.F_T.F_f$$

where K is a numeric constant. F_A is factor corresponding to the geometric parameter 'A' and so on. F_f is the factor corresponding to f_c. For example, the K and F values for maximum negative moment at support are,

$K = -4.712$, $F_A = 1.555 + 0.000787\ (A - 50)$,
$F_B = 1.06 - 0.00022\ (B - 864)$, $F_C = 1.2 + 0.00276\ (C - 864)$,
$F_L = 1.0 + 0.000748\ (L - 2030)$, $F_H = 1.0 + 5.9 \times 10^{-6}\ (H - 2440)$,
$F_T = 0.39 + 0.00173(T - 91)$, $F_f = 1.0$

Thus the value of support negative moment can be readily calculated once the values of A, B etc. are decided. In a similar fashion, expression for other design forces and moments are developed. The equations are presented in Table 1 to facilitate easy calculation.

Comparison of the proposed equations with finite element results: To verify the acceptability of the proposed equations, values given by these equations are compared with the corresponding values obtained from finite element analysis. For the purpose of comparison, five examples are used whose geometries are selected arbitrarily within the scope of the equations. The geometry of these examples are listed in the following table 2. Figure 7(a) through (d) shows the comparison of forces and moments. In all cases the proposed equations give reasonably accurate results on safe side thus establishing their acceptability.

Table 1 Design Worksheet

		A= ___mm. 150 < A < 1000	B=____mm 900 < B < 1875	C=_____mm. 915 < C < 1900	L=_____mm. 2030 < L < 3550	H=_____mm. 2440 < H < 4320	T=_____mm. 100 < T < 280	f'_c=_____MPa. 14 < f'_c < 40	Value
Vertical deflection at landing corner, mm	Constant	$1+0.00545(A-25)^{0.94}$	$1+0.00114(B-914)^{1.1}$	$1+0.00165(C-14)^{0.93}$	$1-7.87\times10^{-4}(L-2030)$	$1-19.68\times10^{-6}(H-2440)$	$1-0.161(T-100)^{0.334}$	$1-1.074\times10^{-6}(f'_c-4)^{.93}$	
	2.03								
Support negative moment, kN-m	Constant	1.555+0.000787(A-	$1.06-0.00022(B-860)$	$1.2+0.00276(C-864)$	$1+0.000748(L-2030)$	$1+5.9\times10^{-6}(H-2440)$	$0.39+0.00173(T-90)$	1.0	
	-4.712								
Flight midspan positive moment, kN-m	Constant	$1.1-31.48\times10^{-4}(A-50)^{1.52}$	$1-70.11\times10^{-4}(B-915)^{1.365}$	1.0	$1+0.128\times10^{-6}(L-2030)^{2.66}$	$1+0.899\times10^{-9}(H-2440)^{2.77}$	$1-.00165(T-100)^{1.17}$	1	
	-1.526								
Negative moment at kink, kN-m	Constant	$1.23+0.000512(A-25)$	$1.01+0.00323(B-915)$	$.85+0.000709(C-915)$	1.0	1.0	$0.95+0.00447(T-100)^{1.03}$	1	
	-3.447								
Negative moment at midsection of landing, kN-m	Constant	$1+0.000303(A-150)$	$1+0.00118(B-915)$	$1+0.00106(C-915)$	$1+0.000409(L-2030)$	1+26.37x10⁻⁴(H-	$1+0.00185(T-100)$	1	
	-6.14								
Axial force in flights, kN	Constant	$1+0.000236(A-125)$	$1+0.000787(B-915)$	$1+0.000827(C-915)$	$1+0.000354(L-2030)$	$1-0.000157(H-2440)$	$1+0.00276(T-100)$	1	
	34.69								
Torsion in flights, kN-m.	Constant	$1+0.00177(A-125)$	$1+0.00063(B-915)$	$1+0.00268(C-915)$	$1-8.0\times10^{-6}(L-2030)^{.75}$	1.0	$1+0.00358(T-100)$	1	
	2.312								
Inplane moment in flights, kN-m	Constant	$1.1+0.000866(A-150)$	$1+0.000984(B-915)$	$1+0.00157(C-915)$	$1+0.00059(L-2030)$	$1-0.000197(H-2440)$	$1+0.0026(T-100)$	1	
	-14.35								
Lateral shear in mid-section of landing, kN	Constant	$1-0.000276(A-150)$	$1+0.00138(B-915)$	$1+0.000709(C-915)$	$1+0.000669(L-2030)$	$1-0.00024(H-2440)$	$1+0.000746(T-100)^{1.3}$	1	
	30.17								

Design of Reinforcement: The equations presented in the previous section give working values of moments and forces. Since elastic analysis is made throughout, these forces and moments are directly proportional to load. To convert from working values to ultimate design values these are to be multiplied by appropriate factors. It is assumed that the conversion factor will be equal to the ratio of factored ultimate load and un-factored service load.

Table 2. Stairway Examples

	Values of geometric parameters, mm					
	A	*B*	*C*	*L*	*H*	*T*
Example 1	300	900	900	2000	2400	100
Example 2	500	1200	1200	2000	2400	100
Example 3	900	1800	1800	3500	4200	200
Example 4	700	1500	1200	2500	3000	150
Example 5	500	1200	1200	2500	3000	150

Apart from maintaining the standard code provisions in detailing reinforcement, there are some important features which are special to the free-standing stairway. The total bending moment at support is not distributed uniformly across the width of the section. This characteristic property of free-standing stair must be taken into account while laying out steel. The outer half of the width of the section carries a greater portion of this moment. On the basis of the results obtained from the parametric study and other examples it is recommended that the outer half of the width of section should be supplied with two-thirds of the total negative steel and the inner half with the rest. Similar proportioning should also be done at flight-landing junction but in reverse order. At midspan of flights, the positive steel is to be distributed uniformly across the section. Of the total steel required to resist the negative bending at mid-landing section, 50 percent should be placed within the inner 1/3 of the width of section. The rest will be distributed across of the outer two-thirds of the width.

The suggested reinforcement layout and bar curtailment scheme for the free-standing stairawy is shown in Fig.8. Half of the negative steel at support may be terminated at a distance of L/4 from the support. Another 25 percent may be bent downward at a distance of L/4 to provide part of the flight midspan positive steel. The rest 25 percent is recommended to continue straight towards the flight-landing junction. This 25 percent may be merged with the negative steel at kink. Fifty percent of the flight midspan positive steel should span from kink and terminate at a point L/5 from the support unless they are bent up for negative steel. The rest should start from a point at a distance of L/5 from kink and will terminate at *L*/4 from support. Of the total negative steel at mid-landing section, half of it will terminate at a distance C/2 from free edge and the rest will cover the whole length of landing. Half of the negative steel at kink will project into landing upto the free edge and the rest may be terminated at a distance of B/2.

DESIGN EXAMPLE

The application of the proposed analysis and design guide is shown here through an example. The input data for design is shown below:

Geometry: A = 300 mm., B= 1220 mm., C= 1220 mm., L= 2540 mm., H = 3050 mm., T= 125 mm., Rise of steps= 150 mm. Here it is to be noted that T is initially assumed.

Loads: Live load = $4.785\text{x}10^{-3}$ MPa (on horizontal projection), floor finish= $0.72\text{x}10^{-3}$ MPa. Dead load is calculated based on unit weight $2.356\text{x}10^{-5}$ N/mm^3.

Material properties: f_c' = 20 MPa, f_y = 275 MPa

Multiplying Factor for ultimate design values: The calculated values of forces and moments are working stress values, which do not incorporate load due to floor finish. These should be multiplied by an appropriate factor so that ultimate design values can be obtained including the effect of floor finish. The procedure is straightforward and is shown below:
The dead loads on horizontal projection can be calculated as,
without floor finish = $5.21\text{x}10^{-3}$ MPa., and with floor finish= $6.05\text{x}10^{-3}$ MPa
Therefore, applying appropriate load factors, the correction factor becomes,

$$\frac{\text{Total load with FF}}{\text{Total load without FF}}=\frac{1.7\times 4.785\times 10^{-3}(\text{LL})+1.4\times 6.05\times 10^{-3}(\text{DL})}{4.785\times 10^{-3}+5.21\times 10^{-3}}\approx 1.7$$

Checking of deflection: Before forces and moments are calculated, it is advisable to check the vertical deflection at the extreme corners of the landing. From the first equation of Table 1 the deflection comes out to be 3.5 mm, which is acceptable.

Checking of thickness requirement: Adequacy of slab thickness should be checked using the maximum bending moment which is, for the present case, the moment at mid landing section which is calculated as 14.84 kN-m based on proposed equations. It has been assumed that the inner one-third of the width of the section carries half of the total moment as suggested earlier. Hence the thickness should be checked accordingly.
Now, half of the ultimate moment $=0.5\times 1.7\times 14.84$ = 12.61 kN-m., one-third of the width = 1220/3 =407 mm. Base on the above moment and width, after necessary calculation, required effective depth d = 75 mm., effective depth provided = 88 mm > 75 mm (OK).

Analyses and reinforcement design: The design moments and forces are calculated using the proposed equations. Necessary reinforcements are calculated afterwards. The entire calculation is shown in a Table 3. The layout of the reinforcement is shown in Fig.9.

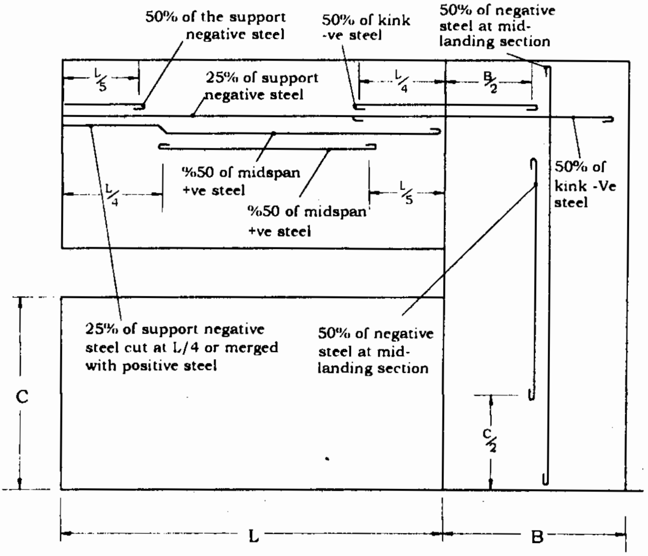


Fig 8. *Suggested bar curtailment and bent details*

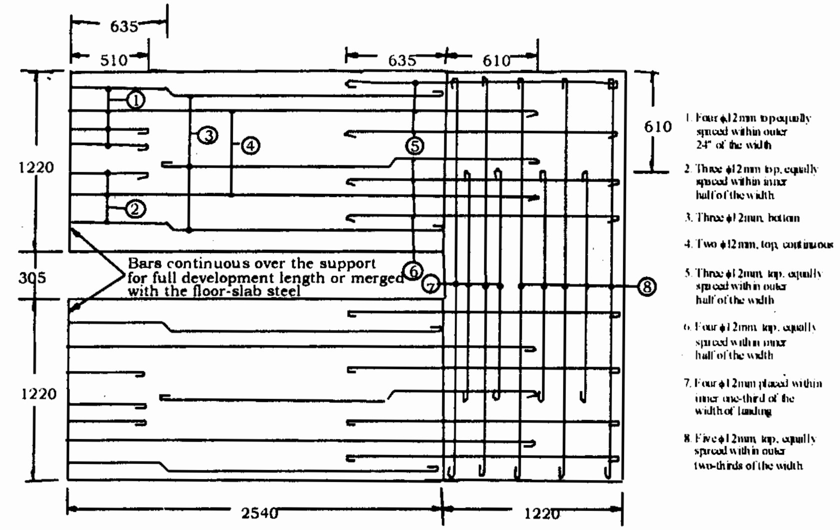


Fig 9. *Reinforcement details of the example stairway*

Support reactions: If support reactions are necessary for the design of the supporting structure, the flight design moments and forces can be taken as design values. These are support bending moment of flights, flight torsion, in-plane moment of flights, flight axial force and the transverse shear in flights. It is to be noted that these values are in the inclined local axes system. It may be necessary to transform them to equivalent values in global axes system prior to design of the supporting structure, which may be a transverse beam or a stiff slab.

Table 3. Analysis and Reinforcement Design Table

Description	**Calculated Values**	**Factored Values**	**Reinforcement**
Support moment	11.07	18.82	A_s = 922 mm^2, provide 12 mm dia bar 7 nos.
Moment a mid-span of flight	3.83	6.51	A_s = 316 mm^2, provide 12 mm dia bar 3 nos.
Moment at kink	10.4	17.68	A_s = 860 mm^2, provide 12 mm dia bar 7 nos.
Moment at mid-landing	14.85	25.23	A_s = 1240 mm^2, provide 12 mm dia bar 10 nos.
Axial tension in upper flight	64	109	A_s = 395 mm^2 , provide 12 mm dia bar 3 nos.
Torsion in flights	7.01	11.92	Closed rectangular stirrups of 10 mm dia bar @ 300 mm c/c. Longitudinal steel, A_l = 435 mm^2.
In-plane moment in flights	41.48	70.52	A_s = 260 mm^2, provide 12 mm dia bar 2 nos.
Lateral shear at mid-landing	60	102	Half of the width of section is effective. Use closed stirrups of 10 mm dia bar @ 280 mm c/c.

CONCLUSIONS

The stairway behaves as a three dimensional plate structure, which is clearly indicated by its deflected shape. Except at mid-span of flights, bending moment at other critical sections is not distributed uniformly across the width of section. Moment is concentrated near the outer edge at support and near the inner edge at kink and at mid landing section. The deflected profile of the stair on horizontal plane clearly indicates that the effects of axial forces in flights (elongation in upper flight and shortening in lower flight) are more than offset by the effect of in-plane moments which causes lateral sway of the whole stair towards the upper flight, Fig.3.

The analytical approaches are not practically suitable for the analysis of free standing stairs so far as economy and efficiency in design are concerned. These methods fail to simulate the actual interaction of plates in three dimension. Also these approaches cannot demonstrate the variation of stress resultants across any cross section.

Based on the present investigation a simple and straightforward method of determining the required design forces and moments has been developed. A reinforcement layout scheme for the stairway has also been suggested taking into account the non-uniform distribution of forces and moments at different critical sections of the stairway. The method is applicable in the most frequently occurring cases. In brief, the main advantages of the proposed method are,

- The required forces and moments can be obtained quickly and easily from the suggested equations without going through any formal analysis. This will relieve the designer from the rigorous calculation required even in the approximate analytical methods. The whole process of analyses can further be simplified by writing separate small computer programs or spreadsheets.
- The required forces and moments produced by the given equations are always on the safe side but within acceptable limits of accuracy.
- The proposed reinforcement layout scheme will optimize the use of reinforcement resulting in a safer structure.
- The resulting proportions of the stair slab structure are optimum. Thus economy is achieved by avoiding over conservative design.

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