

DEFLECTION OF SEMI-RIGIDLY CONNECTED BEAMS

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ABSTRACT: The paper reports theoretical analysis on the serviceability deflection of the semi-rigidly connected beams (concrete, steel and composite) under centre point loading, third point loading and uniformly distributed loading. The semi-rigid beam to column connection has been simulated by spring of variable stiffness. Using moment area theorem and assuming elastic behaviour; equations have been derived for end moment, maximum deflection and its location for different end conditions. The derived equations have been incorporated into EXCEL workbook through which design charts have been prepared for computation of end moments, location and magnitude of maximum deflection. The required minimum depth to span ratio for beams has also been explored.

KEYWORDS: Semi-rigid, connection stiffness, allowable deflection, moment area method.

INTRODUCTION

A beam should be designed in a way so that it is safe against collapse and serviceable in use. Serviceability requires that deflection be adequately small, cracks, if any, should be kept within the tolerable limits. Traditional concept of analysing frame assumes a beam-to-column connection either fully rigid or pin. Numerous tests (Davison *et al* 1990 and Li *et al*, 1996) and numerical studies (Ahmed, B. and Nethercot, D. A., 1995, 1996, 1997, 1998) have proven that the actual behaviour is neither pin nor rigid but rather semi-rigid. Practically almost all the beam-to-column connection exhibits this behaviour i.e., between rigid and pin connection. Deflection is greatly influenced by support condition e.g. simply supported uniformly loaded beam will deflect 5 times than an identical beam with fixed supports. Thus calculating deflection assuming a pin connection at the end will result in a higher estimation of deflection whereas assumption of rigid connection underestimates the deflection. It thus becomes essential to acknowledge the exact or real support condition i.e., the stiffness of the beam-to-column connection for deflection calculation. Thus the problem can be reduced to the estimation of the support stiffness for estimating the deflection. Methods (Ahmed, B. and Nethercot, D. A., 1997; EC3, 1992 and Jones *et al*, 1983) are presently available to predict the connection stiffness and thus it provides an opportunity for

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investigating the beam deflection from a new angle. Presently the maximum deflection of beams are limited by various code requirements, the limits usually expressed in terms of deflection-span ratio. The paper provides an opportunity to explore the required span to depth ratio depending on the load and support condition.

GENERAL EXPRESSION FOR END MOMENT

Since deflection calculation is required at serviceability condition, elastic analysis is adequate for deflection computation. Figure 1 (Ahmed, B. 2002) shows the applied load together with the elastic loading diagram for a simply supported beam with point load at the mid span having support stiffness K_A and K_B at the ends A and B respectively. The span of the beam is L . Where as Fig 2 shows the deflected shape, slope and tangents required for the application of the moment area theorem. From the definition of stiffness,

$$\theta_A = \frac{M_A}{K_A} \text{ and } \theta_B = \frac{M_B}{K_B}$$

Using the moment-area theorem from Figure 2(b) in conjunction with

Fig 1, $\theta_A = \frac{t_{BA}}{L}$

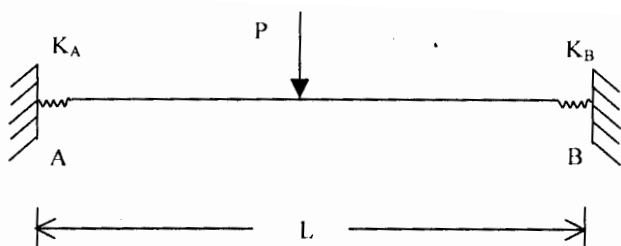
$$\Rightarrow M_A \left(\frac{EI}{K_A} + \frac{L}{3} \right) + \frac{M_B L}{6} - \frac{PL^2}{16} = 0 \quad (1)$$

From Figure 2(a), $\theta_B = \frac{t_{AB}}{L}$

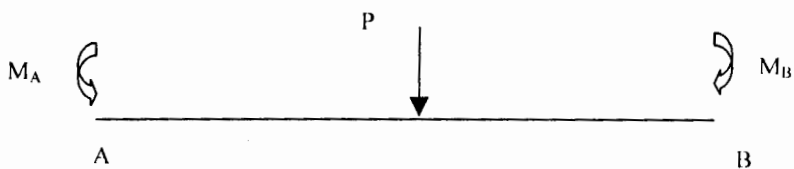
$$\Rightarrow \frac{M_A L}{6} + M_B \left(\frac{EI}{K_B} + \frac{L}{3} \right) - \frac{PL^2}{16} = 0 \quad (2)$$

Solving equations (1) and (2)

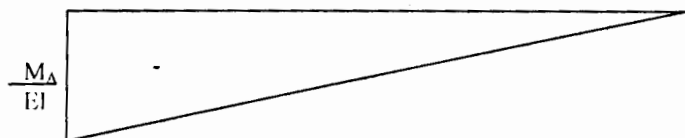
$$M_A = \left(\frac{PL}{8} \right) \frac{\left[1 - \frac{L}{6 \left(\frac{EI}{K_B} + \frac{L}{3} \right)} \right]}{\left(\frac{EI}{K_A} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{K_B} + \frac{L}{3} \right)}} \cdot \frac{L}{2} \quad \text{or } M_A = \left(\frac{PL}{8} \right) f_1 \quad (3)$$



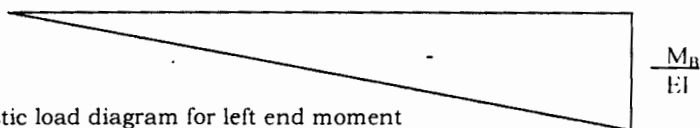
1(a) Beam with centre point loading



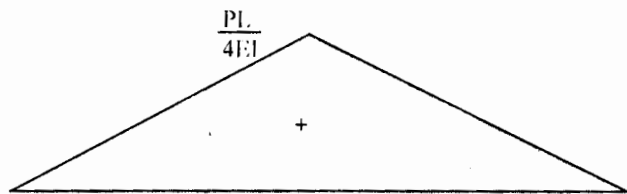
1(b) Free body of the beam



1(c) Elastic load diagram for right end moment



1(d) Elastic load diagram for left end moment



1(e) Elastic load for centre point loading

Fig 1. Semi-rigidly connected beam with centre point loading

$$M_B = \left(\frac{PL}{8}\right) \frac{\left[1 - \frac{L}{6\left(\frac{EI}{K_A} + 3\right)}\right]}{\left(\frac{EI}{K_B} + 3\right) - \frac{L^2}{36\left(\frac{EI}{K_A} + 3\right)}} \frac{L}{2} \text{ or } M_B = \left(\frac{PL}{8}\right) f_2 \quad (4)$$

Equation (3) and (4) give general expression for end moments of beams having different support stiffness.

In a similar way for two pint loading using similar technique:

$$M_A = \left(\frac{2PL}{9}\right) \frac{\left[1 - \frac{L}{6\left(\frac{EI}{k_B} + 3\right)}\right]}{\left(\frac{EI}{k_A} + 3\right) - \frac{L^2}{36\left(\frac{EI}{k_B} + 3\right)}} \frac{L}{2} \text{ or } M_A = \left(\frac{2PL}{9}\right) f_1 \quad (5)$$

$$M_B = \left(\frac{2PL}{9}\right) \frac{\left[1 - \frac{L}{6\left(\frac{EI}{k_A} + 3\right)}\right]}{\left(\frac{EI}{k_B} + 3\right) - \frac{L^2}{36\left(\frac{EI}{k_A} + 3\right)}} \frac{L}{2} \text{ or } M_B = \left(\frac{2PL}{9}\right) f_2 \quad (6)$$

For uniformly distributed loading:

$$M_A = \left(\frac{\omega L^2}{12} \right) \frac{1 - \frac{L}{6 \left(\frac{EI}{k_B} + \frac{L}{3} \right)}}{\left(\frac{EI}{k_A} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{k_B} + \frac{L}{3} \right)}} \frac{L}{2} \text{ or } M_A = \left(\frac{\omega L^2}{12} \right) f_1 \quad (7)$$

$$M_B = \left(\frac{\omega L^2}{12} \right) \frac{\left[1 - \frac{L}{6 \left(\frac{EI}{k_A} + \frac{L}{3} \right)} \right]}{\left(\frac{EI}{k_B} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{k_A} + \frac{L}{3} \right)}} \frac{L}{2} \text{ or } M_B = \left(\frac{\omega L^2}{12} \right) f_2 \quad (8)$$

Thus it can be seen from equations 3 to 8 that the end moments for the semi-rigidly connected beams are equal to the fixed end moment of rigidly connected corresponding beam multiplied by the same function in all load cases. Thus the general equations for end moments can be written as:

$$M_A = M_o \cdot f_1(K_A, K_B) \text{ and } M_B = M_o \cdot f_2(K_A, K_B) \quad (9)$$

M_o is the respective end moment for fixed end condition. At the same time it is of interest to note that the mid span moment can be obtained by:

$$M_{mid-span} = M_{mid-simple} - \frac{1}{2}(M_A + M_B)$$

Where:

$$f_1(K_A, K_B) = \frac{\left[1 - \frac{L}{6 \left(\frac{EI}{k_B} + \frac{L}{3} \right)} \right] \frac{L}{2}}{\left(\frac{EI}{k_A} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{k_B} + \frac{L}{3} \right)}}$$

$$f_1(K_A, K_B) = \frac{6m+n}{4m(3m+n) + (4m+n)} \quad (10)$$

$$f_2(K_A, K_B) = \frac{\left[1 - \frac{L}{6 \left(\frac{EI}{k_A} + \frac{L}{3} \right)} \right] \frac{L}{2}}{\left(\frac{EI}{k_B} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{k_A} + \frac{L}{3} \right)}}$$

$$f_2(K_A, K_B) = \frac{n(6m+1)}{4m(3m+n) + (4m+n)} \quad (11)$$

Here $K_B = n$, K_A and $EI/L = m K_A$.

Figs 3 and 4 show the variation of f_1 and f_2 with m and n . It is thus possible to obtain f_1 and f_2 simply by calculating m and n . It should be noted that the result shown in these two figures represents the following boundary problems: both end variable stiffness from pin to rigid and propped cantilever with right end pinned. The limiting condition that is not covered by these figs is propped cantilever with left end pin connection due to the way of presenting the equation (i.e., the definition of m is $EI/L = m K_A$). To cover this boundary there are two ways: to define m by K_B and produce another set of Figs the other way that is preferable is to use the mirror image of the structure to obtain the solution and than mirror the structure with the obtained solution.

$$f_1(K_A, K_B) = \frac{\left[1 - \frac{L}{6 \left(\frac{EI}{k_B} + \frac{L}{3} \right)} \right] \frac{L}{2}}{\left(\frac{EI}{k_A} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{k_B} + \frac{L}{3} \right)}} \quad (10)$$

$$f_1(K_A, K_B) = \frac{6m + n}{4m(3m + n) + (4m + n)}$$

$$f_2(K_A, K_B) = \frac{\left[1 - \frac{L}{6 \left(\frac{EI}{k_A} + \frac{L}{3} \right)} \right] \frac{L}{2}}{\left(\frac{EI}{k_B} + \frac{L}{3} \right) - \frac{L^2}{36 \left(\frac{EI}{k_A} + \frac{L}{3} \right)}} \quad (11)$$

$$f_2(K_A, K_B) = \frac{n(6m + 1)}{4m(3m + n) + (4m + n)}$$

Here $K_B = n \cdot K_A$ and $EI/L = m \cdot K_A$.

Figs 3 and 4 show the variation of f_1 and f_2 with m and n . It is thus possible to obtain f_1 and f_2 simply by calculating m and n . It should be noted that the result shown in these two figures represents the following boundary problems: both end variable stiffness from pin to rigid and propped cantilever with right end pinned. The limiting condition that is not covered by these figs is propped cantilever with left end pin connection due to the way of presenting the equation (i.e., the definition of m is $EI/L = m \cdot K_A$). To cover this boundary there are two ways: to define m by K_B and produce another set of Figs the other way that is preferable is to use the mirror image of the structure to obtain the solution and than mirror the structure with the obtained solution.

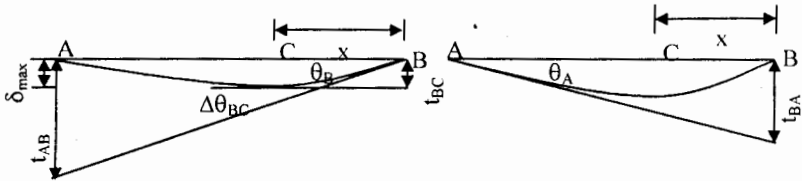


Fig 2. Deflected shape showing tangents and location of maximum deflection

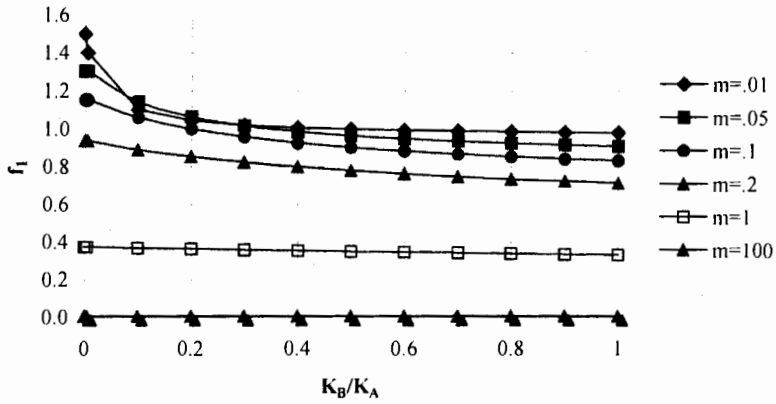


Fig 3. Variation of f_1 with n and m

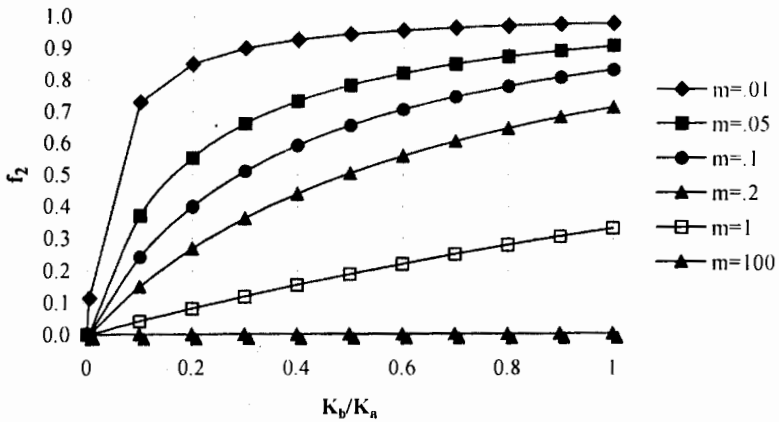


Fig 4. Variation of f_2 with n and m

GENERAL EXPRESSION FOR LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

Centre point load:

Assuming maximum deflection occurs at point C at distance x from the right support (see Figures 1 and 2) and using moment area theorem:

$$\Delta \theta_{BC} = \theta_B - \theta_C$$

Since maximum deflection occurs at point C, $\theta_C = 0$

$$\text{and, } \theta_B = \frac{1}{EI} \left[-\frac{M_A L}{6} - \frac{M_B L}{3} + \frac{PL^2}{16} \right]$$

$$\Delta \theta_{BC} = -\frac{M_A x^2}{2LEI} - \frac{M_B(2Lx - x^2)}{2LEI} + \frac{Px^2}{4EI}$$

Since, $\Delta \theta_{BC} = \theta_B$

$$(-24M_A + 24M_B + 12PL)x^2 - 48M_B Lx + (8M_A L^2 + 16M_B L^2 - 3PL^3) = 0$$

$$\frac{x}{L} = \frac{-f_2 - \sqrt{f_2^2 - \frac{1}{3}(f_1 - f_2 - 4)(3 - 2f_2 - f_1)}}{(f_1 - f_2 - 4)} \quad (12)$$

Substituting values of f_1 and f_2 in equation (12) the location of maximum deflection can be obtained.

From Figures 1 and 2(a),

$$\delta_{\max} = t_{BC}$$

$$\delta_{\max} = -\frac{1}{6EIL} \left[2M_A x^3 + M_B(3Lx^2 - 2x^3) \right] + \frac{Px^3}{6EI}$$

$$\delta_{\max} = -\frac{P}{48EI} \left[2f_1 x^3 + f_2(3Lx^2 - 2x^3) \right] + \frac{Px^3}{6EI} \quad (13)$$

thus:

$$\frac{EI\delta_{\max}}{P} = -\frac{1}{48} \left[2f_1 x^3 + f_2(3Lx^2 - 2x^3) \right] + \frac{x^3}{6} \quad (14)$$

$$\frac{EI\delta_{\max}}{PL^3} = -\frac{1}{48}\left(\frac{x}{L}\right)^2 \left[3f_2 + 2\frac{x}{L}(f_1 - f_2 - 4) \right] \quad (15)$$

From equations 14 and 15 in conjunction with equation 12, it can be seen that $(\delta_{\max}EI/P)$ and $(\delta_{\max} EI)/(PL^3)$ is independent of P and is a function of K_A , K_B and EI/L .

Equation (13) represents the general equation for determining maximum deflection of a beam subjected to a concentrated load applied at the mid-span.

Figure 5 shows the variation of location of maximum deflection with ratio of end stiffness K_b/K_a (n) and beam EI/L to connection stiffness K_a ratio (m) for centre point loading. For any beam-to-column connection it is possible to estimate the end stiffness, thus from the ratio of end stiffness' and beam EI/L to connection stiffness ratio using this figure location of maximum deflection can easily be obtained. Fig 6 shows the variation of $(\delta_{\max} EI)/(PL^3)$ with ratio of end stiffness K_b/K_a (n) and beam EI/L to connection stiffness K_a ratio (m) for centre point loading. Knowing ratio of end stiffness' and beam EI/L to connection stiffness it is possible to obtain $(\delta_{\max} EI)/(PL^3)$ from this figure. Once $(\delta_{\max} EI)/(PL^3)$ is known it can be utilised in several ways like:

- Calculating maximum deflection when the connection stiffnesses, beam section, span and loading is known.
- Calculating the beam section through few trials, when beam span, load and allowable deflection is known.
- Calculating the possible combination of required end stiffness for known beam section, span, load and allowable deflection.

Third point load:

$$3x^2(M_A - M_B) + 2xL(3M_B - PL) + L^2(PL - M_A - 2M_B) = 0$$

$$\frac{x}{L} = \frac{(9 - 6f_2) - \sqrt{(6f_2 - 9)^2 - 12(f_1 - f_2)(4.5 - f_1 - 2f_2)}}{6(f_1 - f_2)} \quad (16)$$

$$\delta_{\max} = -\frac{1}{6EI} [2M_A x^3 + M_B (3Lx^2 - 2x^3)] + \frac{PL}{162EI} (27x^2 - L^2)$$

$$\delta_{\max} = -\frac{P}{27EI} [2f_1 x^3 + f_2 (3Lx^2 - 2x^3)] + \frac{PL}{162EI} (27x^2 - L^2) \quad (17)$$

$$\frac{\delta_{\max} EI}{P} = -\frac{1}{27} [2f_1 x^3 + f_2 (3Lx^2 - 2x^3)] + \frac{L}{162} (27x^2 - L^2) \quad (18)$$

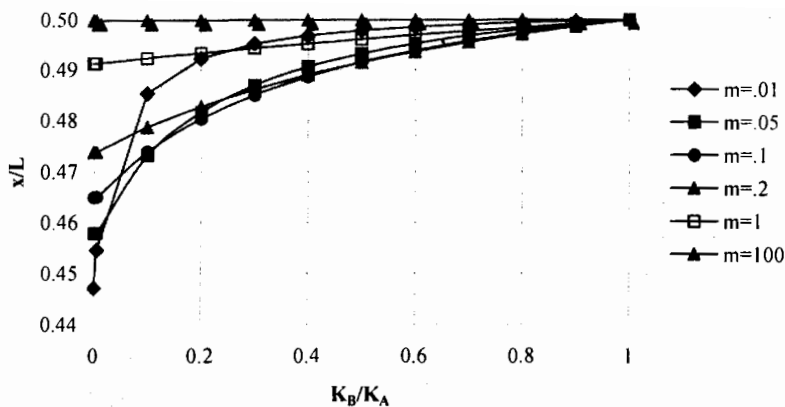


Fig 5. Variation of location of maximum deflection with ratio of end stiffness and m for centre point load

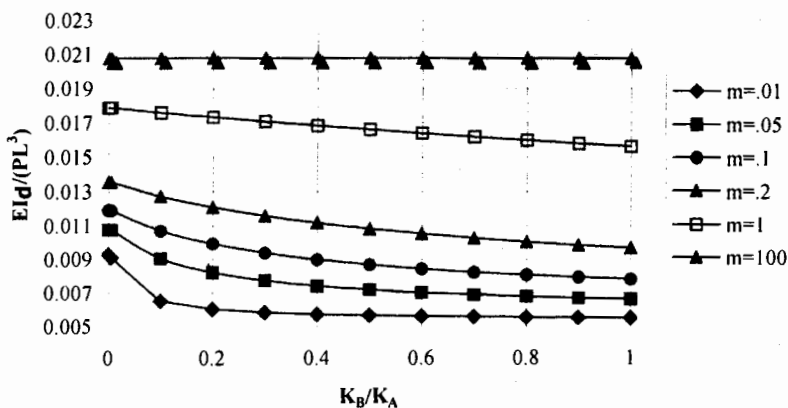


Fig 6. Variation of maximum deflection with ratio of end stiffness and m for centre point load

$$\frac{EI\delta_{\max}}{PL^3} = \frac{1}{54} \left(\frac{x}{L} \right)^2 \left[9 - 4f_1 \frac{x}{L} + 2f_2 \left(2 \frac{x}{L} - 3 \right) \right] - \frac{1}{162} \quad (19)$$

Figs 7 and 8 shows the variation of location of maximum deflection and the variation of $(\delta_{\max} EI)/(PL^3)$ with ratio of end stiffness K_b/K_a (n) and beam EI/L to connection stiffness K_a ratio (m) for third point loading. The use of these figures is same as the previous cases.

Uniformly distributed load:

$$x^3(-8\omega L) + x^2(-12M_A + 12M_B + 8\omega L^2) + x(-24M_B L) + 4M_A L^2 + 8M_B L^2 - \omega L^4 = 0$$

$$24 \left(\frac{x}{L} \right)^3 - 3 \left(\frac{x}{L} \right)^2 (8 + f_2 - f_1) + 6f_2 \left(\frac{x}{L} \right) - (f_1 + 2f_2 - 3) = 0 \quad (20)$$

$$\delta_{\max} = -\frac{1}{6EI} [2M_A x^3 + M_B (3Lx^2 - 2x^3)] + \frac{5}{24EI} \omega x^3 (L - x) \quad (21)$$

$$\frac{EI\delta_{\max}}{\omega} = -\frac{1}{6\omega L} [2M_A x^3 + M_B (3Lx^2 - 2x^3)] + \frac{5}{24} x^3 (L - x) \quad (22)$$

$$\frac{EI\delta_{\max}}{\omega L^4} = \frac{1}{72} \left(\frac{x}{L} \right)^2 \left[15 \frac{x}{L} \left(1 - \frac{x}{L} \right) - 2f_1 \frac{x}{L} - f_2 \left(3 - 2 \frac{x}{L} \right) \right] \quad (23)$$

Figs 9 and 10 shows the variation of location of maximum deflection and the variation of $(\delta_{\max} EI)/(\omega L^4)$ with ratio of end stiffness K_b/K_a (n) and beam EI/L to connection stiffness K_a ratio (m) for uniformly distributed load. The use of these figures is same as the first cases.

It can be seen from equations 13, 17 and 21 that for all the load cases maximum deflection can be generally expressed as:

$$\delta_{\max} = \delta_{s-r} + \delta_{s-s}$$

Where δ_{s-r} is the reduction in deflection due to semi-rigid action of the beam to column connection form the deflection of the pin connected beam and is given by:

$$\delta_{s-r} = -\frac{1}{6EI} [2M_A x^3 + M_B (3Lx^2 - 2x^3)]$$

δ_{s-s} is dependent on the loading type and not directly dependent on the end conditions and is given by:

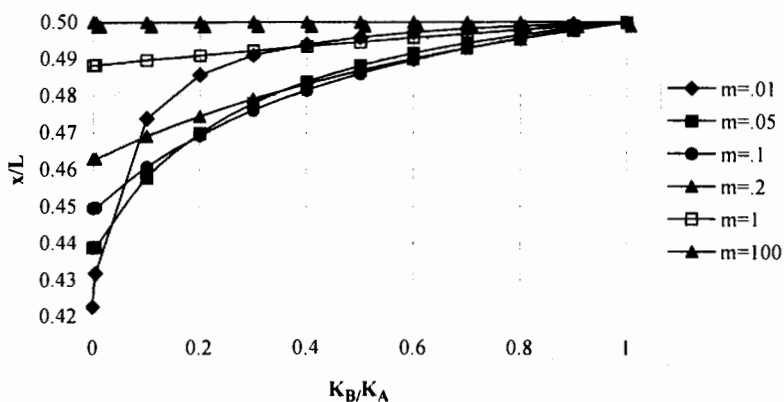


Fig 7. Variation of location of maximum deflection with ratio of end stiffness and m for third point load

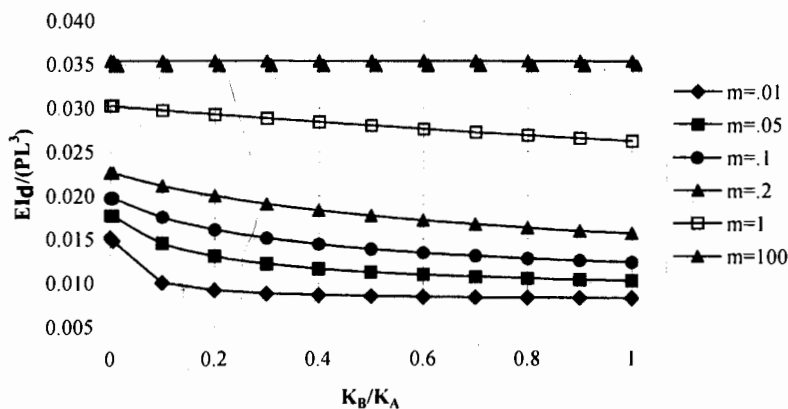


Fig 8. Variation of maximum deflection with ratio of end stiffness and m for third point load

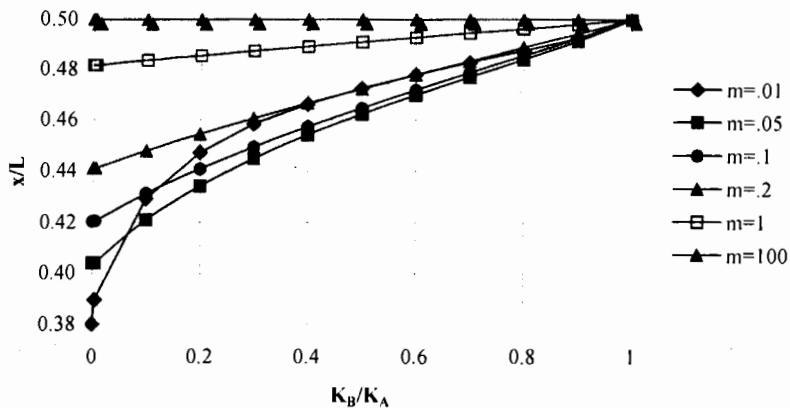


Fig 9. Variation of location of maximum deflection with ratio of end stiffness and m for uniformly distributed load

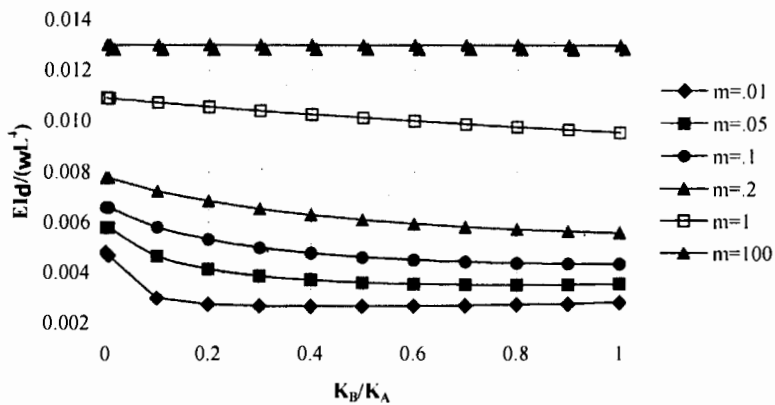


Fig 10. Variation of maximum deflection with ratio of end stiffness and m for uniformly distributed load

$$\delta_{s-x} = \frac{Px^3}{6EI} \quad \text{Centre point load}$$

$$\delta_{s-x} = \frac{PL}{162EI} (27x^2 - L^2) \quad \text{Third point load}$$

$$\delta_{s-x} = \frac{5}{24EI} \omega x^3 (L-x) \quad \text{Uniformly distributed load}$$

It should be noted that knowing the end stiffness at first end moments is to be obtained, using the end moment's location of the maximum deflection is to be computed. Using the end moments and the location of maximum deflection, the magnitude of the maximum deflection can be obtained.

GENERAL EXPRESSION FOR REQUIRED LENGTH TO DEPTH RATIO

Centre point load:

Equation 15 is helpful to compute the required L/d ratio for any beam by substituting a proper equation of I for the section under use. For example using a rectangular section having dimensions bxd,

$$I = \frac{bd^3}{12} \quad \text{and thus for rectangular cross sections:}$$

$$\frac{L}{d} = \left[\frac{1}{\left[-\frac{1}{6PL^4} \{2M_a x^3 + M_b (3Lx^2 - 2x^3)\} + \frac{x^3}{6L^3} \right]} \frac{\delta_{allow} Eb}{12P} \right]^{\frac{1}{3}}$$

$$\frac{L}{d} = \left[\frac{1}{\frac{1}{48} \left(\frac{x}{L} \right)^2 \left[3f_2 + 2 \frac{x}{L} (f_1 - f_2 - 4) \right]} \right]^{\frac{1}{3}} \left(\frac{\delta_{allow} Eb}{12P} \right)^{\frac{1}{3}}$$

$$\frac{L}{d} = f_3 \left(\frac{\delta_{allow} Eb}{12P} \right)^{\frac{1}{3}} \quad (24)$$

Figure 11 shows the variation f_3 with ratio of end stiffness (n) and beam EI/L to connection stiffness ratio (m) for centre point loading.

Using the allowable deflection, selected width of the beam (that is usually governed by architectural demand) and load, L/d can easily be computed from above equation.

Third point load

Like the previous case equation 19 is usable to obtain the required L/d ratio. For rectangular cross sections:

$$\frac{L}{d} = \left(\frac{1}{\frac{1}{54} \left(\frac{x}{L} \right)^2 \left[9 - 4f_1 \frac{x}{L} + 2f_2 \left(2 \frac{x}{L} - 3 \right) \right] - \frac{1}{162}} \right)^{\frac{1}{3}} \left(\frac{\delta_{allow} Eb}{12P} \right)^{\frac{1}{3}}$$

$$\frac{L}{d} = f_4 \left(\frac{\delta_{allow} Eb}{12P} \right)^{\frac{1}{3}} \quad (25)$$

Figure 12 shows the variation f_4 with ratio of end stiffness (n) and beam EI/L to connection stiffness ratio (m) for third point loading. Using the allowable deflection, selected width of the beam (that is usually governed by architectural demand) and load, L/d can easily be computed from above equation.

Uniformly distributed load

Equation 21 can be used to calculate the required L/d ratio. For rectangular cross sections:

$$\frac{L}{d} = \left(\frac{1}{\frac{1}{72} \left(\frac{x}{L} \right)^2 \left[15 \frac{x}{L} \left(1 - \frac{x}{L} \right) - 2f_1 \frac{x}{L} - f_2 \left(3 - 2 \frac{x}{L} \right) \right]} \right)^{\frac{1}{3}} \left(\frac{Eb \delta_{allow}}{\omega} \right)^{\frac{1}{3}}$$

$$\frac{L}{d} = f_5 \left(\frac{Eb \delta_{allow}}{12\omega L} \right)^{\frac{1}{3}} \quad (26)$$

Figure 13 shows the variation f_5 with ratio of end stiffness (n) and beam EI/L to connection stiffness ratio (m) for uniformly distributed load. Using allowable deflection, span, selected width of the beam (that

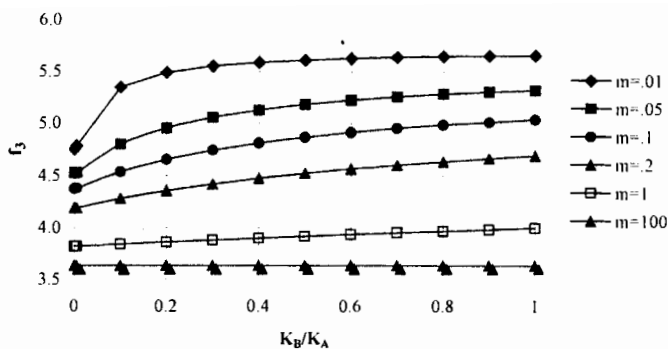


Fig 11. Variation of f_3 with ratio of end stiffness and m for centre point load

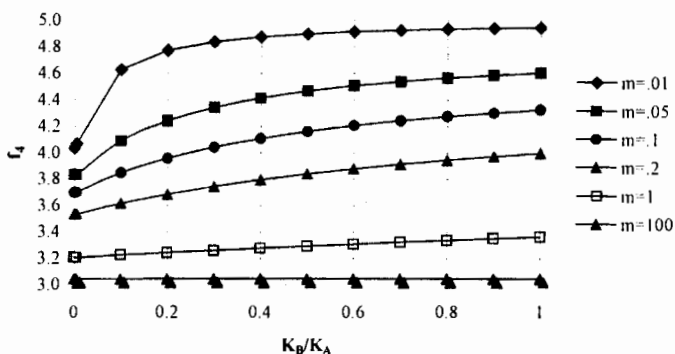


Fig 12. Variation of f_4 with ratio of end stiffness and m for third point load

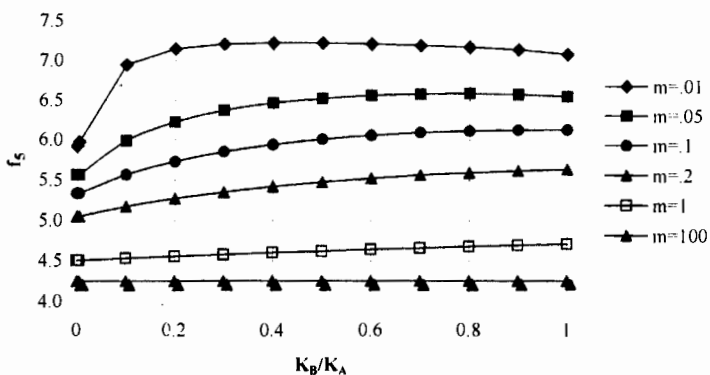


Fig 13. Variation of f_5 with ratio of end stiffness and m for uniformly distributed load

is usually governed by architectural demand) and load, L/d can easily be computed from above equation.

Equation 24 to 26 to be used in conjunction with allowable deflection limits. Thus Figs 11 to 13 provides necessary information to compute the required depth for rectangular beams, considering allowable deflection and the semi-rigid action of the beam-to-column connection. For non-rectangular section, equations 15, 19 and 23 to be used in combination with allowable deflection limits and Figs 3 and 4 to estimate the required span to depth ratio.

DESIGN CHARTS

Figures 3 and 4 provide the coefficients (with respect to connection stiffness ratio K_b/K_a (n) and ratio of beam EI/L to connection stiffness K_a (m)) required for computing the end moments. Figures 5, 7 and 9 provide the location of maximum deflection with respect to connection stiffness ratio K_b/K_a (n) and ratio of beam EI/L to connection stiffness K_a (m). Figures 6, 8 and 10 shows the generalised deflection calculation chart in terms of $(\delta_{max} EI)/(PL^3)$ or $(\delta_{max} EI)/(wL^4)$ with respect to connection stiffness ratio K_b/K_a (n) and ratio of beam EI/L to connection stiffness K_a (m). For known span, load, allowable deflection and preferred ratio of K_b/K_a it is possible to obtain the required beam section.

CONCLUSION

Numerous codes attempt to check the deflection of beams (concrete, steel and composite) by imposing various depth-to-span ratios, but in all the cases the contribution of beam-to-column connection stiffness is neglected. Also from the codes the actual deflection is not known. This paper provides a systematic approach towards the computation of deflection and method of selection of beam section (moment of inertia) and beam-to-column connection stiffness to ensure that the deflection can be kept within the allowable limit.

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